6/1/2013

NORTH SMITHFIELD SCHOOL DEPARTMENT

ALGEBRA 2 CURRICULUM GRADES 10-12

North Smithfield High School
Curriculum Writers: Robin Broman and Thomas Yeaw

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The North Smithfield Mathematics Curriculum for grades K-12 was completed in June 2013 by a K-12 team of teachers. The team, identified as the Mathematics Task Force and Mathematics Curriculum Writers referenced extensive resources to design the document that included:

- *Common Core State Standards for Mathematics*
- *Common Core State Standards for Mathematics, Appendix A*
- *Best Practice, New Standards for Teaching and Learning in America's Schools*
- *Classroom Instruction That Works,* Marzano
- Differentiated Instructional Strategies
- Goals for the district
- High School Traditional Plus Model Course Sequence, Achieve, Inc.
- Khan Academy
- Numerous state curriculum Common Core frameworks, e.g. Ohio Department of Education (ODE), Tucson Unified School District, Arizona (TUSD), New Jersey and Connecticut
- PARCC Model Content Frameworks
- The Illustrative Mathematics Project
- Third International Mathematics and Science TIMSS)
- *Understanding Common Core State Standards,* Kendall

Mission Statement

To foster the success of all students,
our mission is to engage them
in a challenging mathematics curriculum,
driven by standards-based instruction and focused on
mathematical practices, skills, concepts, and problem solving.

The North Smithfield Mathematics Curriculum identifies what students should know and be able to do in mathematics. Each grade or course includes Common Core State Standards (CCSS), grade level Assessment problems, teacher notes, best practice instructional strategies, resources, a map (or suggested timeline), rubrics, checklists, and common formative and summative assessments.

COMMON CORE STATE STANDARDS

The **Common Core State Standards** (**CCSS):**

- Are fewer, higher, deeper, and clearer.
- Are aligned with college and workforce expectations.
- Include rigorous content and applications of knowledge through high-order skills.
- Build upon strengths and lessons of current state standards (GLEs and GSEs).
- Are internationally benchmarked, so that all students are prepared for succeeding in our global economy and society.
- Are research and evidence-based.

Common Core State Standards components include:

- Standards for **Mathematical Practice** (K-12)
- Standards **for Mathematical Content:**
    - Categories (high school only): e.g. numbers, algebra, functions, data
    - Domains: larger groups of related standards
    - Clusters: groups of related standards
    - Standards**:** define what students should understand and are able to do

The **North Smithfield Common Core Mathematics Curriculum** provides all students with a sequential comprehensive education in mathematics through the study of:

- Standards **for Mathematical Practice (K-12)**
    - Make sense of problems and persevere in solving them
    - Reason abstractly and quantitatively
    - Construct viable arguments and critique the reasoning of others
    - Model with mathematics*
    - Use appropriate tools strategically
    - Attend to precision
    - Look for and make use of structure
    - Look for and express regularity in repeated reasoning

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- Standards **for <u>Mathematical Content</u>:**
    - **K – 5 Grade Level Domains of**
        - Counting and Cardinality
        - Operations and Algebraic Thinking
        - Number and Operations in Base Ten
        - Number and Operations – Fractions
        - Measurement and Data
        - Geometry
    - **6-8 Grade Level Domains of**
        - Ratios and Proportional Relationships
        - The Number System
        - Expressions and Equations
        - Functions
        - Geometry
    - **9-12 Grade Level Conceptual Categories of**
        - Number and Quantity
        - Algebra
        - Functions
        - Modeling
        - Geometry
        - Statistics and Probability

RESEARCH-BASED INSTRUCTIONAL STRATEGIES

The North Smithfield Common Core Mathematics Curriculum provides a list of research-based **best practice instructional strategies** that the teacher may model and/or facilitate. It is suggested the teacher:

- Use **formative assessment to** guide instruction
- Use ***Classroom Instruction That Works*** (Marzano)
    - Setting objectives and providing feedback
    - Reinforcing effort and providing recognition
    - Cooperative learning
    - Cues, questions, and advance organizers
    - Nonlinguistic representations
    - Summarizing and note taking
    - Assigning homework and providing practice
    - Identifying similarities and differences
    - Generating and testing hypotheses
- Provide opportunities for **independent, partner** and **collaborative group work**
- Differentiate **instruction** by varying the **content, process, and product** and providing opportunities for:
    - anchoring
    - cubing
    - jig-sawing
    - pre/post assessments
    - tiered assignments
- Address **multiple intelligences** instructional strategies, e.g. visual, bodily kinesthetic, interpersonal
- Provide opportunities for **higher level thinking: Webb's Depth of Knowledge, 2,3,4,** skill/conceptual understanding, strategic reasoning, extended reasoning
- Facilitate the integration of **Mathematical Practices** in all content areas of mathematics

- Facilitate integration of the **Applied Learning Standards (SCANS):**
  - communication
  - critical thinking
  - problem solving
  - reflection/evaluation
  - research
- Employ strategies of "best practice" (student-centered, experiential, holistic, authentic, expressive, reflective, social, collaborative, democratic, cognitive, developmental, constructivist/heuristic, and challenging)
- Provide **rubrics** and **models**
- Address **multiple intelligences** and brain dominance (spatial, bodily kinesthetic, musical, linguistic, intrapersonal, interpersonal, mathematical/logical, and naturalist)
- Employ **mathematics best practice strategies** e.g.
  - using manipulatives
  - facilitating cooperative group work
  - discussing mathematics
  - questioning and making conjectures
  - justifying of thinking
  - writing about mathematics
  - facilitating problem solving approach to instruction
  - integrating content
  - using calculators and computers
  - facilitating learning
  - using assessment to modify instruction

COMMON ASSESSMENTS

The North Smithfield Common Core Mathematics Curriculum includes common assessments. Required (red ink) indicates the assessment is required of all students e.g. common tasks/units**,** standardized mid-term exam, standardized final exam.

- **REQUIRED** COMMON ASSESSMENTS
  - MID-TERM EXAM
  - FINAL EXAM
  - COMMON PROBLEMS/UNITS
- **Common Instructional Assessments (I)** - used by teachers and students during the instruction of CCSS.
- **Common Formative Assessments (F)** - used to measure how well students are mastering the content standards **before** taking state assessments
  - teacher and student use to make decisions about what actions to take to promote further learning
  - on-going, dynamic process that involves far more frequent testing
  - serves as a practice for students
- **Common Summative Assessment (S)** - used to measure the level of student, school, or program success
  - make some sort of judgment, e.g. what grade
  - program effectiveness
  - e.g. state assessments (AYP), mid-year and final exams
- Additional suggested assessments include:
  - Anecdotal records
  - Conferencing
  - Exhibits
  - Interviews
  - Graphic organizers
  - Journals
  - Mathematical Practices
  - Modeling
  - Multiple Intelligences assessments, e.g.
    - Role playing - bodily kinesthetic
    - Graphic organizing - visual
    - Collaboration - interpersonal
  - Oral presentations
  - Problem/Performance based/common tasks
  - Rubrics/checklists (mathematical practice, modeling)
  - Tests and quizzes
  - Technology
  - Think-alouds
  - Writing genres
    - Argument
    - Informative
    - Research

RESOURCES FOR ALGEBRA 2

Textbooks
- *Algebra 2,* McDougal Littell
- *Exploration in Core Math* , Holt Mc Dougal

Supplementary

Technology
- Computer lab
- Computer software that generate graphs of functions
- Computers
- Document camera
- Graphing calculator
- Graphing software
- Interactive boards
- LCD projectors
- Overhead graphing scientific
- SMART Boards
- Student response systems

Websites
- http://curriculum.northsmithfieldschools.com
- http://www.achieve.org/http://my.hrw.com
- http://www.illustrativemathematics.org/standards/practice
- http://www.ixl.com/standards/common-core/math/grade-8
- http://www.ixl.com/standards/common-core/math/high-school
- http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDefaultPage.aspx?page=1
- http://www.ode.state.or.us/search/page/?id=3747
- http://www.parcconline.org/sites/parcc/files/PARCC%20Math%20S
- http://www.schools.utah.gov/CURR/mathsec/Core.aspx
- http://www.tusd1.org/contents/distinfo/curriculum/index.asp
- www.commoncore.org/maps
- www.corestandards.org
- www.khanacademy.com
- www.ride.ri.gov

Materials
- Hands-on materials, such as algebra tiles
- Tables, graphs and equations of real-world applications that apply quadratic and exponential functions

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ALGEBRA 2 CURRICULUM Grades 10-12

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CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
NUMBER AND QUANTITY **The Complex Number System (N-CN)** Perform arithmetic operations with complex numbers. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others **4.** Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	A	**Students** N-CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. **Additional content** **Essential questions** • *What is a complex number? Why are complex numbers useful?* **Essential knowledge and skills** • The complex number i is defined by the relation $i = \sqrt{-1}$, thus $i^2 = -1$ • Every complex number can be written in the form a + bi where a and b are real numbers. • The square root of a negative number is a complex number. **Teaching Examples** $\sqrt{-1} = i$ $\sqrt{-4} = 2i$ $\sqrt{-7} = \sqrt{7}i$ (TUSD) **Mathematical Practices** • Reason abstractly and quantitatively • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *Before introducing complex numbers, revisit simpler examples demonstrating how number systems can be seen as "expanding" from other number systems in order to solve more equations. For example, the equation x + 5 = 3 has no solution as a whole numbers, but it has a solution x = -2 as an integers. Similarly, although 7x = 5 has no solution in the integers, it has a solution x = $\frac{5}{7}$ in the rational numbers. The linear equation ax + b = c, where a, b, and c are rational numbers, always has a solution x in the rational numbers:* $x = \frac{(c-b)}{a}$. *).* • *When moving to quadratic equations, once again some equations do not have solutions, creating a need for larger number systems. For example, $x^2 - 2 = 0$ has no solution in the rational numbers. But it has solutions $\pm\sqrt{2}$ in the real numbers. (The real number line augments the rational numbers, completing the line with the irrational numbers.)* • *Point out that solving the equation $x^2 - 2 = 0$ in terms of x is equivalent to finding x-intercepts of a graph of $y = x^2 - 2$, which crosses the x-*	RESOURCE NOTES See resources in the introduction Textbook • *Algebra 2*, McDougal Littell • *Exploration in Core Math* , Holt Mc Dougal Supplementary Books, Teacher (T) Student (S) Technology • Computers • Graphing calculator • Interactive boards • LCD projectors Websites • http://curriculum.northsmithfieldschools.com http://www.achieve.org /http://my.hrw.com • http://www.illustrativemathematics.org/standards/practice • http://www.ixl.com/standards/common-core/math/grade-8 • http://www.ixl.com/standards/common-core/math/high-school • http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDefaultPage.aspx?page=1 http://www.ode.state.or.us/search/page/?id=3747 • http://www.parcconline.org/sites/parcc/files/PARCC%20Math%20S • http://www.schools.utah.gov/CURR/mathsec/C	ASSESSMENT NOTES See assessments in the introduction REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS • Anecdotal records • Charts/data collection • Conferencing • Exhibits • Interviews • Graphic organizers • Journals • Mathematical Practices • Modeling ★ • Multiple Intelligences assessments, e.g. ▫ Role playing - bodily kinesthetic ▫ Graphic organizing - visual
	A	N-CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **Additional content** **Essential questions** • *How can you add, subtract, and multiply complex numbers?* **Essential knowledge and skills** • Complex numbers can be added, subtracted, and multiplied like binomials. • The commutative, associative, and distributive properties hold true when adding, subtracting, and multiplying complex numbers. **Teaching Examples** • Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3-2i)(-7+4i)$ **Mathematical Practices** • Reason abstractly and quantitatively • Use appropriate tools strategically • Attend to precision • Look for and make use of structure •			

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		$(3-2i)(-7+4i)$ $3(-7+4i)-2i(-7+4i)$ Distributive Property $-21+12i+14i-8i^2$ Distributive Property $-21+(12i+14i)-8i^2$ Associative Property $-21+i(12+14)-8i^2$ Distributive Property $-21+26i-8i^2$ Computation $-21+26i-8(-1)$ $i^2=-1$ $-21+26i+8$ Computation $-21+8+26i$ Commutative Property $-13+26i$ Computation (TUSD) **Academic vocabulary** • Associative property • Commutative property • Complex number system • Conjugate pair • Distributive property • Fundamental Theorem of Algebra • Index • Linear factors • Polynomial • Radical • Radicand • Real number system • Root • Solution • Standard form (a + bi) • Zero of a polynomial ASSESSMENT PROBLEMS N-CN.1 Basic • http://www.purplemath.com/modules/complex.htm (#1-4) • http://www.shmoop.com/common-core-standards/ccss-hs-n-cn-1.htm • http://alex.state.al.us/lesson_view.php?id=11364 (Classifying Complex Numbers.ppt (Resource); Categorizing Complex Numbers Test Mode.ppt (Resource);Categorizing Complex Numbers.ppt (Resource) N-CN.2 Basic • http://www.purplemath.com/modules/complex.htm (#5-8)	*axis at $+\sqrt{2}$) and -$\sqrt{2}$ √). Thus, the graph illustrates that the solutions are √ .* • *Next, use an example of a quadratic equation with real coefficients, such as $x^2 + 1 = 0$, which can be written equivalently as $x^2 = -1$. Because the square of any real number is non-negative, it follows that $x^2 = -1$ has no solution in the real numbers. One can see this graphically by noticing that the graph of $y = x^2 + 1$ does not cross the x-axis.* • *The "solution" to this "impasse" is to introduce a new number, the imaginary unit i, where $i^2 = -1$, and to consider complex numbers of the form a +bi, where a and b are real numbers and i is not a real number. Because i is not a real number, expressions of the form a +bi cannot be simplified.* • *The existence of i, allows every quadratic equation to have two solutions of the form a + bi – either real when b = 0, or complex when b ≠ 0. Have students observe that if a quadratic equation (with real coefficients) has complex solutions, the solutions always appear in conjugate pairs, in the form a + bi and a – bi. Particularly, for an equation $x^2 = - 9$, a conjugate pair of solutions are 0 +3i and 0 – 3i.* (ODE)	ore.aspx • http://www.tusd1.org/contents/distinfo/curriculum/index.asp www.commoncore.org/maps • www.corestandards.org • www.khanacademy.com • www.ride.ri.gov Materials • Hands-on materials, such as algebra tiles • Graphic calculators	▫ Collaboration - interpersonal • Oral presentations • Problem/Performance based/common tasks • Rubrics/checklists (mathematical practice, modeling) • Tests and quizzes • Technology • Think-alouds • Writing genres ▫ Argument ▫ Informative ▫ Research

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CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
NUMBER AND QUANTITY **The Complex Number System (N-CN)** Use complex numbers in polynomial identities and equations. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	A	**Students** N-CN.7 Solve quadratic equations with real coefficients that have complex solutions. **Additional content** __Essential questions__ • *How can you solve a quadratic equation that has no real zeros?* • __Essential knowledge and skills__ • Quadratic equations can have real and complex solutions. • All quadratic polynomials have two roots. • Complex roots of quadratics occur in conjugate pairs __Teaching Examples__ This standard has a direct connection to the standard HS.A.REI.4 in the Algebra conceptual category • Within which number system can $x^2 = -2$ be solved? Explain how you know. • Solve $x^2 + 2x + 2 = 0$ over the complex numbers. • Find all solutions of $2x^2 + 5 = 2x$ and express them in the form a + bi. (TUSD) __Mathematical Practices__ • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure N-CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. __Essential questions__ • Will a quadratic equation with real coefficients always have real solutions? Why or why not? __Essential knowledge and skills__ • Polynomial identities allow us to rewrite polynomials using complex numbers. __Teaching Examples__ • Polynomial identities include the quadratic formula, factoring quadratic expressions, the difference of two squares, and the sum and difference of two cubes. Example: • Use the difference of two squares to rewrite $x^2 +$ 4. • Solution: $x^2 + 4 = x^2 – (–4) = (x + 2i)(x – 2i)$. (TUSD) __Mathematical Practices__ • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *Revisit quadratic equations with real coefficients and a negative discriminant and point out that this type of equation has no real number solution. Emphasize that with the extension of the real number system to complex numbers any quadratic equation has a solution. Since the process of solving a quadratic equation may involve the use of the quadratic formula with a negative discriminant, defining a square root of a negative number becomes critical* $\sqrt{-N} = i\sqrt{N}$ *where N is a positive real number; i is the imaginary unit and* $i^2 = -1$*). After the square root of a negative number has been defined, emphasize that the quadratic formula can be used without restriction.* • *While solving quadratic equations using the quadratic formula, students should observe that the quadratic equation always has a pair of solutions regardless of the value of the discriminant. If the discriminant,* $b^2 – 4ac$*, is positive, the equation has two unequal complex solutions that are real (the imaginary parts of complex numbers are zeros). If the discriminant is zero, the*	RESOURCE NOTES See resources in the introduction	ASSESSMENT NOTES __REQUIRED COMMON ASSESSMENTS__ • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS __SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS__ See assessments in the introduction

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<table>
<tr><th>CATEGORIES, DOMAINS, CLUSTERS</th><th>UNIT</th><th>STANDARDS/BENCHMARKS
North Smithfield School Department</th><th>INSTRUCTIONAL STRATEGIES</th><th>RESOURCES</th><th>ASSESSMENTS</th></tr>
<tr><td></td><td></td><td>N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

<u>Essential questions</u>
• How can you tell how many roots a polynomial has?
• How can you show that any quadratic will have two roots?
<u>Essential knowledge and skills</u>
• The Fundamental Theorem of Algebra tells us how many roots a polynomial has; some of the roots may be complex numbers.
<u>Teaching Examples</u>
• The Fundamental Theorem of Algebra states that a polynomial of degree n has n roots (zeros). Some of the roots may be complex, and some roots may be the same.
Examples:
• How many zeros does $-2x^2+3x-8$ have? Find all of the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra.
• How many complex zeros does the following polynomial have? How do you know?
$p(x)=(x^2-3)(x^2+2)(x-3)(2x-1)$
(TUSD)

<u>Mathematical Practices</u>
• Reason abstractly and quantitatively
• Model with mathematics ★
• Use appropriate tools strategically
• Attend to precision
• Look for and make use of structure

<u>Common Student Misconceptions</u>
• Students fail to understand the complex number system. When adding, subtracting, and multiplying complex numbers, the method is similar to those of polynomials. Therefore, students simply view the “i” as a variable rather than as a specific number.
• Students fail to understand the cyclical pattern for the powers of i.
• Students fail to understand that polynomial equations always have solutions, some of which may be complex numbers. Students tend to stop solving a quadratic if the discriminant is negative and “write no real number solution” as the solution.
• Students fail to understand that a quadratic can be factored using complex solutions. Given x2 + 4 = 0, students will believe that the quadratic cannot factor since does not look like a difference of 2 squares. They need to be able to factor x2 + 4 as (x – 2i)(x + 2i).
• Students fail to connect the behavior of the graph with the roots of the polynomials. There is a disconnect between the x-intercepts of the graph and the roots of the polynomial, especially when the polynomial has complex roots.

<u>Academic vocabulary</u>
• Associative property
• Commutative property
• Complex number system
• Conjugate pair
• Fundamental Theorem of Algebra
• Index
• Linear factors
• Radicand
• Real number system
• Root
• Solution</td><td>equation has a repeated real solution – a double root (two complex solutions with equal real parts and the imaginary parts equal to zero). If the discriminant is negative, the equation has two complex conjugate solutions that are not real.
(ODE)</td><td></td><td></td></tr>
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CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• Distributive property • Polynomial • Radical • Standard form (a + bi) • Zero of a polynomial ASSESSMENT PROBLEMS N-CN.7 Basic • http://www.shmoop.com/common-core-standards/ccss-hs-n-cn-7.html (Math.N-CN.7 Quiz) N-CN.8 Advanced • http://www.shmoop.com/common-core-standards/ccss-hs-n-cn-8.html N-CN.9 Advanced • http://www.shmoop.com/common-core-standards/ccss-hs-n-cn-9.html			
ALGEBRA **Seeing structure in Expressions (A-SSE)** Interpret the structure of expressions. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning		**Students** A-SSE.1 Interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors, and coefficients. (A-SSE.1a) b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (A-SSE.1b) ○ *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **Essential questions** • Give an example of a real-world problem and write an expression to model the relationship. Explain how the algebraic symbols represent the words in the problem. • How are coefficients and factors related to each other? • How does viewing a complicated expression by its single parts help to interpret and solve problems? • What does it mean to call something a quantity? **Essential knowledge and skills** • Expressions consist of terms (parts being added or subtracted). • Terms can either be a constant, a variable with a coefficient or a variable raised to a power. • Real-world problems with changing quantities can be represented by expressions with variables. • The relationship between the abstract symbolic representations of expressions can be identified based on how they relate to the given situation. • Complicated expressions can be interpreted by viewing parts of the expression as single entities. **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *Polynomial and rational* • *Extending beyond simplifying an expression, this cluster addresses interpretation of the components in an algebraic expression. A student should recognize that in the expression 2x + 1, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as "2x" and "1" being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content.* • *Using real-world context examples, the nature of algebraic expressions can be explored. For example, suppose the cost of cell phone service for a month is represented by the expression 0.40s + 12.95. Students can analyze how*	RESOURCE NOTES See resources in the introduction • Hands-on materials, such as algebra tiles, can be used to establish a visual understanding of algebraic expressions and the meaning of terms, factors and coefficients	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

This curriculum was developed based on the Common Core State Standards utilizing examples and strategies from various websites including Tucson, Arizona, Ohio, and New Jersey.

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
	M	**Teaching Examples** In Algebra I, students work with linear, exponential, and quadratic expressions. In Algebra II, students extend these concepts to general polynomials and rational expressions. • Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context. Examples: • What are the factors of $P(1+r)^n$? Which part(s) of this expression depend on P? a. A mixture contains A liters of liquid fertilizer in 10 liters of water. Write an expression for the concentration of fertilizer in the mixture, and explain what each part of the expression represents. b. Another mixture contains twice as much fertilizer in the same amount of water as the mixture in part (a). Write an expression for the concentration of the new mixture, and explain why this concentration is not twice as much as the concentration of the first mixture. (TUSD) A-SSE.2 Use the structure of an expression to identify ways to rewrite it. • *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **Major content** **Essential questions** • *How does using the structure of an expression help to simplify the expression?* • *Why would you want to simplify an expression*? **Essential knowledge and skills** • Structure within expressions can be identified and used to factor or simplify the expression. **Teaching Examples** • Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. Examples: • Factor $x^3 - 2x^2 - 35x$ **Mathematical Practices** • Reason abstractly and quantitatively • Use appropriate tools strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning	*the coefficient of 0.40 represents the cost of one minute (40¢), while the constant of 12.95 represents a fixed, monthly fee, and s stands for the number of cell phone minutes used in the month. Similar real-world examples, such as tax rates, can also be used to explore the meaning of expressions.* • *Factoring by grouping is another example of how students might analyze the structure of an expression. To factor 3x(x – 5) + 2(x – 5), students should recognize that the "x – 5" is common to both expressions being added, so it simplifies to (3x + 2)(x – 5). Students should become comfortable with rewriting expressions in a variety of ways until a structure emerges.* • *Have students create their own expressions that meet specific criteria (e.g., number of terms factorable, difference of two squares, etc.) and verbalize how they can be written and rewritten in different forms. Additionally, pair/group students to share their expressions and rewrite one another's expressions.* (ODE) • *Polynomial and rational*		

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		• Factor $x^4 - y^4$ (TUSD) ASSESSMENT PROBLEMS A-SSE.1 Basic • http://www.illustrativemathematics.org/illustrations/531 (Delivery trucks) • http://www.illustrativemathematics.org/illustrations/1343 (Delivery trucks) • http://www.illustrativemathematics.org/illustrations/89 (Increasing or Decreasing) • http://www.illustrativemathematics.org/illustrations/215 (Kitchen Floor Tiles) • http://www.illustrativemathematics.org/illustrations/389 (Mixing Candies) • http://www.illustrativemathematics.org/illustrations/88 (Mixing Fertilizer) • http://www.illustrativemathematics.org/illustrations/187 (Quadrupling Leads to Halving) • http://www.illustrativemathematics.org/illustrations/1366 (Radius of a Cylinder) • http://www.illustrativemathematics.org/illustrations/390 A-SSE.A.1.b (The Bank Account) • http://www.illustrativemathematics.org/illustrations/436 (An Integer Identity) A-SSE.1 Advanced • http://www.illustrativemathematics.org/illustrations/23 (The Physics Professor) • http://www.illustrativemathematics.org/illustrations/90 (Throwing Horseshoes) A-SSE.2 Basic • http://www.illustrativemathematics.org/illustrations/436 (An Integer Identity) • http://www.illustrativemathematics.org/illustrations/919 (Animal Populations) • http://www.illustrativemathematics.org/illustrations/87 (Equivalent Expressions) • http://www.illustrativemathematics.org/illustrations/198 (Sum of Even and Odd) • http://www.illustrativemathematics.org/illustrations/617 N-CN.A (Computations with Complex Number)			
ALGEBRA **Seeing structure in Expressions (A-SSE)** Write expressions in equivalent forms to solve problems. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of		**Students** A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **Supporting content** a. Factor a quadratic expression to reveal the zeros of the function it defines. (A-SSE.3a) b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A-SSE.3b) c. Use the properties of exponents to transform expressions for exponential functions. ○ *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* (A-SSE.3c)	TEACHER NOTES See instructional strategies in the introduction • *Quadratic and exponential* • *This cluster focuses on linking expressions and functions, i.e., creating connections between multiple representations of functional relations – the dependence between a quadratic expression and a graph of the quadratic*	RESOURCE NOTES See resources in the introduction	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	M	**Essential knowledge and skills** • Define and use zero and negative exponents. • Exponential growth and decay formulas • Relate the algebraic and graphic solutions to a quadratic equation (x-intercepts, zero, roots) by ○ Factoring ○ Completing the square ○ Greatest common factor **Teaching Examples:** • Express $2(x^3 – 3x^2 + x – 6) – (x – 3)(x + 4)$ in factored form and use your answer to say for what values of x the expression is zero. • Write the expression below as a constant multiplied by a power of x and use your answer to decide whether the expression gets larger or smaller as x gets larger. $\frac{(2x^3)^2(3x^4)}{(x^2)^3}$ (TUSD) **Mathematical Practices** • Model with mathematics A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. **Major content** ○ *For example, calculate mortgage payments.* ★ **Essential questions** • How can you derive the formula for the sum of a geometric series? • Why must the common ratio in a geometric series be different from 1 in order to use the formula for the sum? **Essential knowledge and skills** • A geometric series is the sum of terms in a geometric sequence. • The sum of a finite geometric series with common ratio not equal to 1 can be written as a simple formula. • Geometric series can be used to solve real-world problems. **Teaching Examples** • In February, the Bezanson family starts saving for a trip to Australia in September. The Bezansons expect their vacation to cost \$5375. They start with \$525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip? (TUSD) **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning	*function it defines, and the dependence between different symbolic representations of exponential functions. Teachers need to foster the idea that changing the forms of expressions, such as factoring or completing the square, or transforming expressions from one exponential form to another, are not independent algorithms that are learned for the sake of symbol manipulations. They are processes that are guided by goals (e.g., investigating properties of families of functions and solving contextual problems).* • *Factoring methods that are typically introduced in elementary algebra and the method of completing the square reveals attributes of the graphs of quadratic functions, represented by quadratic equations.* ○ *The solutions of quadratic equations solved by factoring are the x – intercepts of the parabola or zeros of quadratic functions.* ○ *A pair of coordinates (h, k) from the general form $f(x) = a(x – h)^2 + k$ represents the vertex of the parabola, where h represents a horizontal shift and k represents a vertical shift of the parabola $y = x^2$ from its original position at the origin.* ○ *A vertex (h, k) is the*		

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		Academic vocabulary • Arithmetic sequence • Arithmetic series • Coefficient • Common factor • Conjugates • Constant • Difference of squares • Expression • Factor • Finite series • Geometric sequence • Geometric series • Real number system • Term **Common Student Misconceptions** • Students confuse expressions with equations. When asked to simplify an expression, many students will set the expression equal to 0 and solve it. • Students often do not use the order of operations correctly as they simplify an expression. For example, in the expression , they may incorrectly multiply P to (1 + r) prior to raising (1 + r) to the nth power. (TUSD) ASSESSMENT PROBLEMS A-SSE.3 Basic • Quadratic equations: Solve a quadratic equation by factoring (Algebra - BB.5) • Quadratic equations: Complete the square (Algebra - BB.6) • Exponents: Negative exponents (Algebra - V.3) • Exponents: Multiplication with exponents (Algebra - V.4) • Exponents: Division with exponents (Algebra - V.5) • Exponents: Multiplication and division with exponents (Algebra - V.6) • Exponents: Power rule (Algebra - V.7) • Exponents: Simplify expressions involving exponents (Algebra - V.8) • Algebra review: Properties of exponents (Geometry - A.3) • http://www.schools.utah.gov/CURR/mathsec/Core/Secondary-II/II-3-A-SSE-3.aspx • http://www.ode.state.or.us/wma/teachlearn/commoncore/mat.hs.sr.1.0asse.e.015_v1.pdf A-SSE.4 Basic • http://www.illustrativemathematics.org/illustrations/805 (Course of Antibiotics) • http://www.illustrativemathematics.org/illustrations/442 (Triangle Series)	*minimum point of the graph of the quadratic function if $a > 0$ and is the maximum point of the graph of the quadratic function if $a < 0$. Understanding an algorithm of completing the square provides a solid foundation for deriving a quadratic formula.* • *Translating among different forms of expressions, equations and graphs helps students to understand some key connections among arithmetic, algebra and geometry. The reverse thinking technique (a process that allows working backwards from the answer to the starting point) can be very effective. Have students derive information about a function's equation, represented in standard, factored or general form, by investigating its graph.* (ODE)		
ALGEBRA **Arithmetic with polynomials and rational function (A-APR)** Perform arithmetic operations on polynomials		**Students** A- APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Essential questions** • What is the result when you add, subtract, or multiply polynomials? Is this always true? Why or why not? • How is the system of polynomials similar to and different from the system of integers? **Mathematical Practices** • Reason abstractly and quantitatively • Use appropriate tools strategically	TEACHER NOTES See instructional strategies in the introduction • *The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to*	RESOURCE NOTES See resources in the introduction	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE

This curriculum was developed based on the Common Core State Standards utilizing examples and strategies from various websites including Tucson, Arizona, Ohio, and New Jersey.

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning		• How does the distributive property show that you can combine like terms? • Explain how the distributive property is used to multiply any size polynomials. • Create your own example of adding, subtracting or multiplying two polynomials, where one polynomial is a quadratic, and explain how you would simplify the expression. • Attend to precision • Look for and make use of structure **<u>Essential knowledge and skills</u>** • Adding, subtracting and multiplying two polynomials will yield another polynomial, thus making the system of polynomials closed. • Addition and subtraction of polynomials is combining like terms. • The distributive property proves why you can combine like terms. • Multiplication of polynomials is applying the distributive property. **<u>Teaching Examples</u>** • Simplify: a. $(3x^5 +7x^2 - x-19)+(7x^5-4x^3-2x^2 +6x-3)$ b. $(2x^4-5x^2+3x-7)-(3x^4+x^3-4x^2+2x+5)$ c. $(2x-3)(x^2-3x-5)$ (TUSD) **<u>Academic vocabulary</u>** • Binomial theorem • Closed set • Coefficient • Combinations • Complex solution • Degree • Denominator • Distributive property • Factoring • Inspection method • Multiplicity • Numerator • Pascal's triangle • Polynomials • Rational expression • Remainder theorem • Synthetic division • Zeros ASSESSMENT PROBLEMS A-APR.1 Basic • http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-1.html • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_1.pdf • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_1_ans.pdf • http://map.mathshell.org/materials/download.php?fileid=834 • http://map.mathshell.org/materials/download.php?fileid=835	*work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term.* • *In arithmetic of polynomials, a central idea is the distributive property, because it is fundamental not only in polynomial multiplication but also in polynomial addition and subtraction. With the distributive property, there is little need to emphasize misleading mnemonics, such as FOIL, which is relevant only when multiplying two binomials, and the procedural reminder to "collect like terms" as a consequence of the distributive property. For example, when adding the polynomials 3x and 2x, the result can be explained with the distributive property as follows: $3x + 2x = (3 + 2)x = 5x$.* • *An important connection between the arithmetic of integers and the arithmetic of polynomials can be seen by considering whole numbers in base ten place value to be polynomials in the base b = 10. For two-digit whole numbers and linear binomials, this connection can be illustrated with area models and algebra tiles. But the connections between*		<u>ASSESSMENTS</u> See assessments in the introduction

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			methods of multiplication can be generalized further. For example, compare the product 213 x 47 with the product. $(2b^2+1b+3)(4b+7)$: $\begin{array}{r} 2b^2 + 1b + 3 \\ \times \quad 4b + 7 \\ \hline 14b^2 + 7b + 21 \\ 8b^3 + 4b^2 + 12b \quad \\ \hline 8b^3 + 18b^2 + 19b + 21 \end{array}$ $\begin{array}{r} 200 + 10 + 3 \\ \times \quad 40 + 7 \\ \hline 1400 + 70 + 21 \\ 8000 + 400 + 120 \quad \\ \hline 8000 + 1800 + 190 + 21 \end{array}$ $\begin{array}{r} 213 \\ \times \; 47 \\ \hline 1491 \\ 8520 \\ \hline 10011 \end{array}$ (ODE)		
ALGEBRA **Arithmetic with polynomials and rational function (A-APR)** Understand the relationship between zeros and factors of polynomials. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them	**M**	**Students** A- APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. **Major content** **Essential questions** • *How can you determine whether x – a is a factor of a polynomial p(x)? Why does this work?* • *How do you determine how many zeros a polynomial function will have?* • *Extension: Why is it true that p(x)/(x – a) has a remainder of p(a)?* **Essential knowledge and skills** • The Remainder theorem says that if a polynomial p(x) is divided by x-a , then the remainder is the value of the polynomial evaluated at a. **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *As discussed for the previous cluster (Perform arithmetic operations on polynomials), polynomials can often be factored. Even though polynomials (i.e., polynomial expressions) can be explored as mathematical objects without consideration of functions, in school mathematics, polynomials are usually taken to define functions. Some equations*	RESOURCE NOTES See resources in the introduction Graphing calculators	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

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2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning			• Saying that x – a is a factor of a polynomial p(x) is equivalent to saying that p(a) = 0, by the zero property of multiplication. • Any polynomial of degree n can be factored into n binomials of the form x – c, with possibly complex values for c. • If *p(a)* = 0, then a is a zero of *p*. **Teaching Examples** The Remainder theorem says that if a polynomial p(x) is divided by x – a for some number a, then the remainder is the constant p(a). That is, $p(x) = q(x)(x - a) + p(a)$. So if p(a) = 0, then p(x) = q(x)(x – a). Example: Let $p(x) = x^5 - 3x^4 + 8x^2 - 9x + 30$. Evaluate p(–2). What does your answer tell you about the factors of p(x)? • Solution: p(–2) = 0, so x + 2 is a factor of p(x). (TUSD)	*may include polynomials on one or both sides. The importance here is in distinction between equations that have solutions, and functions that have zeros. Thus, polynomial functions have zeros. This cluster is about the relationship between the factors of a polynomial, the zeros of the function defined by the polynomial, and the graph of that function. The zeros of a polynomial function are the x-intercepts of the graph of the function.* • *Through some experience with long division of polynomials by x-a students get a sense that the quotient is always a polynomial that is one degree less than the degree of the original polynomial, and that the remainder is always a constant. In other words,* $p(x) = q(x)(x - a) + r$. *Using this equation, students reason that p(a)=r . Thus, if p(a) = 0, then the remainder r = 0 , the polynomial p(x) is divisible by (x-a) and (x-a) is a factor of p(x) . Conversely, if (x-a) is a factor of p(x) , then p(a)=0.* (ODE)		
	M	A- APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **Major content** **Essential questions** • *What information do you need to sketch a rough graph of a polynomial function?* • *How are the zeros of a polynomial related to its graph?* **Essential knowledge and skills** • If a is a zero of p, then a is an x-intercept of the graph of y = p(x). • The values and multiplicity of the zeros of a polynomial, along with the end behavior, can be used to sketch a graph of the function defined by the polynomial. **Teaching Examples** Graphing calculators or programs can be used to generate graphs of polynomial functions. Examples: • Factor the expression $x^3 + 3x^2 - 49x - 147$ and explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x) = x^3 + 3x^2 - 49x - 147$. **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure			

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• Factor the expression $x^3 + 4x^2 - 59x - 126$ and explain how your answer can be used to solve the equation $x^3 + 4x^2 - 59x - 126 = 0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x) = x^3 + 4x^2 - 59x - 126$. (TUSD) **Academic Vocabulary** • Binomial theorem • Closed set • Coefficient • Combinations • Complex solution • Degree • Denominator • Distributive property • Factoring • Inspection method • Multiplicity • Numerator • Pascal's triangle • Polynomials • Rational expression • Remainder theorem • Synthetic division • Zeros ASSESSMENT PROBLEMS A-APR.2 Basic • http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-2.html • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_2.pdf • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_2_ans.pdf • http://www.illustrativemathematics.org/illustrations/592 (The Missing Coefficient) • http://www.illustrativemathematics.org/illustrations/788 (Zeroes and factorization of a general polynomial) • http://www.illustrativemathematics.org/illustrations/796 (Zeroes and factorization of a non polynomial) • http://www.illustrativemathematics.org/illustrations/787 (Zeroes and factorization of a quadratic polynomial) • http://www.illustrativemathematics.org/illustrations/789 (Zeroes and factorization of a quadratic polynomial II) A-APR.3 Basic • http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-3.html (Shmoop standard page) • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_3.pdf (Arithmetic with Polynomials – Worksheet 3) • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_3_ans.pdf (Arithmetic with Polynomials – Answers)			

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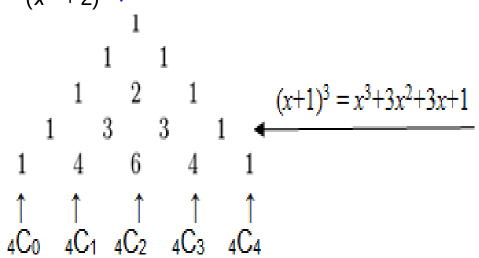
CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
ALGEBRA **Arithmetic with polynomials and rational function (A-APR)** Use polynomial identities to solve problems. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**A**	**Students** A- APR.4 Prove polynomial identities and use them to describe numerical relationships. **Additional content** ○ For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. <u>**Essential questions**</u> • *Where do the coefficients of the terms in a binomial expansion come from? Why does the formula work?* • *Why do the signs of terms alternate when you expand $(x - y)^n$?* <u>**Essential knowledge and skills**</u> • Polynomial identities can be used to describe numerical relationships. <u>**Teaching Examples**</u> • Use the polynomial identity $(x^2+y^2)^2 = (x^2– y^2)^2 + (2xy)^2$ to generate Pythagorean triples. • Use the distributive law to explain why $x^2 – y^2 = (x – y)(x + y)$ for any two numbers x and y. • Derive the identity $(x – y)^2 = x^2 – 2xy + y^2$ from $(x + y)^2 = x^2 + 2xy + y^2$ by replacing y by –y. • Use an identity to explain the pattern ○ $2^2 – 1^2 = 3$ ○ $3^2 – 2^2 = 5$ ○ $4^2 – 3^2 = 7$ ○ $5^2 – 4^2 = 9$ • Solution: $(n + 1)^2 - n^2 = 2n + 1$ for any whole number *n*. (TUSD) <u>**Mathematical Practices**</u> • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure A- APR.5 (+) Know and apply the Binomial Theorem for the expansion of (x+ y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal’s Triangle.1 <u>**Essential questions**</u> • *What is the connection between the Binomial theorem and Pascal's triangle? Why?* • *How can you use the Binomial theorem to solve problems?* <u>**Essential knowledge and skills**</u> • A binomial raised to a power such as $(x + y)^n$ can be expanded into a sum of terms using the Binomial <u>**Mathematical Practices**</u> • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make	TEACHER NOTES See instructional strategies in the introduction • *In Grade 6, students began using the properties of operations to rewrite expressions in equivalent forms. When two expressions are equivalent, an equation relating the two is called an identity because it is true for all values of the variables. This cluster is an opportunity to highlight polynomial identities that are commonly used in solving problems. To learn these identities, students need experience using them to solve problems.* • *Students should develop familiarity with the following special products:* $(x+y)^2 = x^2 + 2xy + y^2$ $(x-y)^2 = x^2 - 2xy + y^2$ $(x+y)(x-y) = x^2 - y^2$ $(x+a)(x+b) = x^2 + (a+b)x + ab$ $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$ • *Students should be able to prove any of these identities. Furthermore, they should develop sufficient fluency with the first four of these so that they can recognize expressions of the form on either side of these identities in order to replace that expression with an equivalent expression in the form of the other side of the*	RESOURCE NOTES See resources in the introduction Graphing calculators	ASSESSMENT NOTES <u>REQUIRED COMMON ASSESSMENTS</u> • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS <u>SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS</u> See assessments in the introduction

This curriculum was developed based on the Common Core State Standards utilizing examples and strategies from various websites including Tucson, Arizona, Ohio, and New Jersey.

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		theorem. use of structure • The coefficients of the terms in a binomial expansion can be found using combinatorics. • Pascal's triangle can be used to find the coefficients of the terms in a binomial expansion. **Teaching Examples** • The Binomial theorem can be proved by mathematical induction or by a combinatorial argument. Examples: • Use Pascal's Triangle to expand the expression $(2x-1)^4$. • Find the middle term in the expansion of $(x^2+2)^{18}$.  • This cluster has many possibilities for optional enrichment, such as relating the example in A-APR.4 to the solution of the system u2+v2=1, v = t(u+1), relating the Pascal triangle property of binomial coefficients to $(x+y)^{n+1} = (x+y)(x+y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction. (TUSD) **Academic vocabulary** • Binomial theorem • Closed set • Coefficient • Combinations • Complex solution • Degree • Denominator • Distributive property • Factoring • Inspection method • Multiplicity • Numerator • Pascal's triangle • Polynomials • Rational expression • Remainder theorem • Synthetic division • Zeros ASSESSMENT PROBLEMS A- APR.4 Basic • http://www.illustrativemathematics.org/illustrations/594 (Trina's Triangles)	*identity.* ○ *With identities such as these, students can discover and explain facts about the number system. For example, in the multiplication table, the perfect squares appear on the diagonal. Diagonally, next to the perfect squares are "near squares," which are one less than the perfect square. Why?* ○ *Why is the sum of consecutive odd numbers beginning with 1 always a perfect square?* ○ *Numbers that can be expressed as the sum of the counting numbers from 1 to n are called triangular numbers. What do you notice about the sum of two consecutive triangular numbers? Explain why it works.* ○ *The sum and difference of cubes are also reasonable for students to prove. The focus of this proof should be on generalizing the difference of cubes formula with an emphasis toward finite geometric series.* (ODE)		

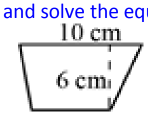
CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_4.pdf (A-REI.4 Worksheet) • http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_4_ans.pdf (A-REI.4 Worksheet) • http://hopehigh.enschool.org/ourpages/auto/2013/1/2/67161447/2-15-13quadratic%20formula.pdf (A-REI.4 b) A- APR.5 Advanced • http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-5.html (Shmoop standard page) • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_5.pdf (Arithmetic with Polynomials – Worksheet 5) • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_5_ans.pdf (Arithmetic with Polynomials – Worksheet 5 Answers)			
ALGEBRA **Arithmetic with polynomials and rational function (A-APR)** Rewrite rational expressions. .. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**S**	**Students** A- APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **Supporting content** **Essential questions** • *How can you write a rational expression in different forms?* • *Why might it be useful to write a rational expression in different forms?* • *When you rewrite $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, why should the degree of r(x) be less than the degree of b(x)?* **Essential knowledge and skills** • A rational expression is a quotient of two polynomials; the denominator must be nonzero. • Rational expressions can be written in different forms using factoring and arithmetic operations. • An expression of the form $\frac{a(x)}{b(x)}$ can be written as $q(x)+\frac{r(x)}{b(x)}$, where a(x), b(x), q(x), and r(x) are polynomials, and the degree of r(x) is less than the degree of b(x). • Inspection and long division are two methods for rewriting a rational expression. **Teaching Examples** • *The limitations on rational functions apply to the* **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES *See instructional strategies in the introduction* • *This cluster is the logical extension of the earlier standards on polynomials and the connection to the integers. Now, the arithmetic of rational functions is governed by the same rules as the arithmetic of fractions, based first on division.* • *In particular, in order to write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, students need to work through the long division described for A-APR.2-3. This is merely writing the result of the division as a quotient and a remainder. For example, we can rewrite $\frac{75}{8}$ in the form $9+\frac{3}{8}$. Note that the fraction $\frac{75}{8}$ is interpreted as*	RESOURCE NOTES See resources in the introduction Graphing calculator	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		rational expressions in A-APR.6. • The polynomial *q(x)* is called the quotient and the polynomial *r(x)* is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. Examples: • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. • Express $f(x)=\frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph. • Solution: $f(x)=\frac{2x+1}{x-1}=\frac{2(x-1)+3}{x-1}=\frac{2(x-1)}{x-1}+\frac{3}{x-1}=2+\frac{3}{x-1}$, so the horizontal asymptote is y = 2. (TUSD) A- APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **Essential questions** • *Why does the denominator of a rational expression have to be nonzero?* • *What is the result when you add, subtract, multiply, or divide rational expressions? Explain how you know.* • *How is the system of rational expressions similar to and different from the system of rational numbers?* **Mathematical Practices** • Reason abstractly and quantitatively • Use appropriate tools strategically • Attend to precision • Look for and make use of structure **Essential knowledge and skills** • Adding, subtracting, multiplying, and dividing rational expressions result in another rational expression, thus making it a closed system. • Adding, subtracting, multiplying, and dividing rational expressions follow the same rules as operations on rational numbers. **Teaching Examples** A-APR.7 requires the general division algorithm for polynomials. Examples:	*the division , so that 75 is the dividend and 8 is the divisor. The result indicates that 9 is the quotient and 3 is the remainder. Note that for division of integers, we expect the remainder to be between 0 and the divisor, which in this case is 8. (If the remainder were greater than or equal to 8, we could subtract another 8, and increase the quotient by 1.)* • *In order to rewrite simple rational expressions in different forms, students need to understand that the rules governing the arithmetic of rational expressions are the same rules that govern the arithmetic of rational numbers (i.e., fractions). To add fractions, we use a common denominator:* $\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$ • *as long as* $b,d \neq 0$. *Although in simple situations, a, b, c, and d would each be whole numbers, in fact they can be polynomials. So now suppose that , a = 2, b = (x-1), c=x, and d= (x+1), then* $\frac{2}{x-1}+\frac{x}{x+1}=\frac{2(x+1)}{(x-1)(x+1)}+\frac{(x-1)x}{(x-1)(x+1)}=\frac{2(x+1)+(x-1)x}{(x-1)(x+1)}$ • *And then the numerator can be simplified further:*		

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• Use your knowledge about the sum of two fractions to explain why the sum of two rational expressions is another rational expression. • Express $\frac{1}{x^2+1} - \frac{1}{x^2-1}$ in the form $\frac{a(x)}{b(x)}$, where *a(x)* and *b(x)* are polynomials in standard form. (TUSD) **Academic vocabulary** • Binomial theorem • Closed set • Coefficient • Combinations • Complex solution • Degree • Denominator • Distributive property • Factoring • Inspection method • Multiplicity • Numerator • Pascal's triangle • Polynomials • Rational expression • Remainder theorem • Synthetic division • Zeros **Common Student Misconceptions** • Students commonly do not understand the definition of a closed set and how it applies to polynomial addition, subtraction, and multiplication. • Students confuse expressions with equations. When asked to simplify an expression, many students will set the expression equal to 0 and solve it. • Students fail to understand the connection between zeros and factors of a polynomial. For example, for the expression 2x2 – 7x + 3, if x = 3, then the expression equals 0. This means that x – 3 must be a factor of the expression. Some students will mistakenly write that 3 is a factor, or that x + 3 is a factor. • When using the Binomial Theorem, students do not apply the power to the coefficients and tend to apply the power only to the variable. For example, when expanding (2x – y)3, students will write 3C3(2x3)(-y0) instead of 3C3(2x)3(-y)0. • Students do not correctly identify the least common denominator when adding or subtracting rational expressions. • When simplifying rational expressions, students do not include the restriction of the domain in their answer. For example, when reducing , students write without including the statement x≠1. This mistake will lead to an error in geometric understanding of the graph. Without the domain restriction, students will not show a hole in the graph at x = 1. (TUSD) ASSESSMENT PROBLEMS A- APR.6 Basic • http://www.illustrativemathematics.org/illustrations/825 (Combined fuel efficiency) • http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-6.html (Shmoop standard page) • http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_6.pdf (Arithmetic with Polynomials – Worksheet 6) http://www.shmoop.com/common-core-standards/handouts/a-arp_worksheet_6_ans.pdf (Arithmetic with Polynomials – Worksheet 6 Answers)	$= \frac{2x+2+x^2-x}{(x-1)(x+1)} = \frac{x^2+x+2}{(x-1)(x+1)}$ (ODE)		

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		A- APR.7 Basic • http://www.shmoop.com/common-core-standards/ccss-hs-a-apr-7.html • http://ww.ixl.com/math/algebra-1/add-and-subtract-rational-expressions • http://ww.ixl.com/math/algebra-1/multiplu-anddivide-rational-expressions			
ALGEBRA **Creating Equations ★ (A-CED)** Create equations that describe numbers or relationships (A-CED) Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**S**	**Students** A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **Supporting content** **Essential questions** • *How do you translate from real-world situations into mathematical equations and inequalities?* • *How do you determine if a situation is best represented by an equation, an inequality, a system of equations or a system of inequalities?* • *Why would you want to create and equation or inequality to represent a real-world problem?* **Essential knowledge and skills** • Equations and inequalities can be created to represent and solve real-world and mathematical problems. **Teaching Examples** • For A-CED.1, use all available types of functions to create such equations, including root functions, but constrain to simple cases. • Equations can represent real-world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. Examples: • Given that the following trapezoid has area 54 cm2, set up an equation to find the length of the unknown base, and solve the equation.  • Lava coming from the eruption of a volcano follows a parabolic path. The height h in feet of a piece of lava t seconds after it is ejected from the volcano is **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *Equations using all types of expressions, including simple root functions* • *Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.* • *Discuss the importance of using appropriate labels and scales on the axes when representing functions with graphs.* • *Examine real-world graphs in terms of constraints that are necessary to balance a mathematical model with the real world context. For example, a student writing an equation to model the maximum area when the perimeter of a rectangle is 12 inches should recognize that $y = x(6 – x)$ only makes sense when $0 < x < 6$. This restriction on the domain is necessary because the side of a rectangle under these conditions cannot be less*	RESOURCE NOTES See resources in the introduction	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

This curriculum was developed based on the Common Core State Standards utilizing examples and strategies from various websites including Tucson, Arizona, Ohio, and New Jersey.

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		given by $h(t) = -16t^2 + 64t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet? (TUSD) A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **Essential questions** • *How are graphs of equations and inequalities similar and different?* **Essential knowledge and skills** • Relationships between two quantities can be represented through the creation of equations in two variables and graphed on coordinate axes with labels and scales. **Teaching Examples** • While functions used in A-CED.2, 3, and 4 will often be linear, exponential, or quadratic, the types of problems should draw from more complex situations than those addressed in Algebra 1. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. Examples: • Find a formula for the volume of a single-scoop ice cream cone in terms of the radius and height of the cone. Rewrite your formula to express the height in terms of the radius and volume. Graph the height as a function of radius when the volume is held constant. • Find the distance from the point (-2, 5) to the line y = 3x + 1. (TUSD) **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. • *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **Essential questions** • *How do you determine if a given point is a viable solution to a system of equations or inequalities, both on a graph and using the equations?* **Mathematical Practices** • Make sense of problems and persevere in solving	*than or equal to 0, but must be less than 6. Students can discuss the difference between the parabola that models the problem and the portion of the parabola that applies to the context.* • *Explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula.* • *For example, the formula for the area of a trapezoid* ($A = \frac{1}{2}h(b_1 + b_2)$) *can be solved for h if the area and lengths of the bases are known but the height needs to be calculated. This strategy of selecting a different representation has many applications in science and business when using formulas .* • *Provide examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations on a graphing calculator to determine which parts of the graph are relevant to the problem context.* • *Use a graphing calculator to demonstrate how dramatically the shape of a curve can change when the scale of the graph is altered for one or both variables.* • *Give students formulas, such as area and volume (or from science or business), and have students solve the equations for each of the different variables in the formula.* (ODE)		

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department		INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		Essential knowledge and skills • Solutions are viable or not in different situations depending upon the constraints of the given context. **Teaching Examples** Example: • A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8. ○ Write a system of inequalities to represent the situation. ○ Graph the inequalities. ○ If the club buys 150 hats and 100 jackets, will the conditions be satisfied? ○ What is the maximum number of jackets they can buy and still meet the conditions? (TUSD)	them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	• *Equations using all types of expressions, including simple root functions*		
		A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. • *For example, rearrange Ohm's law V = IR to highlight resistance R.* **Essential questions** • *Why would you want to solve a given formula for a particular variable?* • *How do you solve a given formula for a particular variable?* **Essential knowledge and skills** • Formulas can be rearranged and solved for a given variable using the same reasoning as in solving an equation. **Teaching Examples** Examples: • The Pythagorean theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$. • Why might the theorem need to be solved for c? • Solve the equation for c and write a problem situation where this form of the equation might be useful.	**Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure			

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS

- Solve $V = \frac{4}{3}\pi r^3$ for radius r.
- Motion can be described by the formula below, where t = time elapsed, u = initial velocity, a = acceleration, and s = distance traveled:
- $s = ut + \frac{1}{2}at^2$
- Why might the equation need to be rewritten in terms of a?
- Rewrite the equation in terms of *a*. (TUSD)

Academic vocabulary

- Axes
- Constraints
- Dependent
- Equations
- Exponential
- Independent
- Inequalities
- Labels
- Linear
- Origin
- Quadratic
- Scales
- Viable solutions

Common Student Misconceptions

- Students often confuse which variable is independent and which is dependent. In addition, students are unable to write an equation that represents the relationship given contextual or geometric information.
- Students do not check for viable solutions. Although students may solve an equation correctly, they don't check for the validity of the solution, especially if it represents an application.
- Students do not define constraints when equations or inequalities represent real-world problems. Students do not consider any restrictions on the domain when solving an equation or inequality.
- Students do not understand how the scale of the axes can affect how we see the graph. A poor window or choice of scale markings on a graph may lead to a misunderstanding of the behavior of the graph and failure to see all solutions to equations or inequalities.
- Students do not understand that the axes can represent variables other than x and y. Students will have difficulties with application problems when the variables are no longer x and y but, for example, t for time and h for height.
- Students have difficulties with equations with multiple unknowns when the equation is to be solved for a different variable in general terms. For example, students will have a harder time solving for W in P = 2W + 2L. However, given specific values for P and L, students can solve for W.

ASSESSMENT PROBLEMS

A-CED.A.1 Basic

- http://www.illustrativemathematics.org/illustrations/702 (Basketball)
- http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_OneVariableWritingEquations.xml (linear)
- http://www.illustrativemathematics.org/illustrations/582 (linear)

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• http://www.illustrativemathematics.org/illustrations/580 ((quadratic) • http://www.illustrativemathematics.org/illustrations/437 • http://www.illustrativemathematics.org/illustrations/702 A-CED.A.1 Advanced • http://www.illustrativemathematics.org/illustrations/83 (linear) A-CED.A.2 Basic • http://www.illustrativemathematics.org/illustrations/1010 (linear) A-CED.A.3 Basic • http://www.illustrativemathematics.org/illustrations/1010 (linear) • http://www.illustrativemathematics.org/illustrations/220			
ALGEBRA **Reasoning with Equations and Inequalities (A-REI)** Understand solving equations as a process of reasoning and explain the reasoning. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	- **M** **M**	**Students** A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **Major content** **Essential questions** • Give an example of a simple rational or radical equation that has an extraneous solution and explain why it is an extraneous solution. **Essential knowledge and skills** • Simple rational and radical equations can have extraneous solutions. **Teaching Examples** Examples: • Solve for x: ○ $\sqrt{x+2}=5$ ○ $\frac{7}{8}\sqrt{2x-5}=21$ ○ $\frac{x+2}{x+3}=2$ ○ $\sqrt{3x-7}=-4$ (TUSD) **Mathematical Practices** • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Use appropriate tools strategically • Attend to precision • Look for and make use of structure A-REI.4 Solve quadratic equations in one variable. **Major content** a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. A-REI.4a	TEACHER NOTES See instructional strategies in the introduction • *Simple radical and rational* • *Challenge students to justify each step of solving an equation. Transforming 2x - 5 = 7 to 2x =12 is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.* • *Provide examples for how the same equation might be solved in a variety of ways as long as equivalent quantities are added or subtracted to both sides of the equation, the order of steps taken will not matter.* $3n + 2 = n - 10$ $-\quad 2 = \quad -2$ $3n \quad = n – 12$ $-n \quad = -n$ $2n \quad = -12$ $n = -6$	RESOURCE NOTES See resources in the introduction Graphing calculators	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS

STANDARDS/BENCHMARKS

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A-REI.4b

Essential knowledge and skills

- Solving quadratic equations by a variety of methods including: inspecting, graphing, taking square roots, factoring, completing the square, quadratic formula.
- Determine the best method for solving quadratic equation.
- Determine why some quadratic equations have extraneous and/or complex solutions.

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2 - 4ac = 0$	1 real root	intersects x-axis once
$b^2 - 4ac > 0$	2 real roots	intersects x-axis twice
$b^2 - 4ac < 0$	2 complex roots	does not intersect x-axis

(TUSD)

Mathematical Practices

- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics ★
- Use appropriate tools strategically
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

ASSESSMENT PROBLEMS

A-REI.2 Basic

- http://www.illustrativemathematics.org/illustrations/702 (Basketball)
- http://www.illustrativemathematics.org/illustrations/391 (Radical Equations)
- http://www.shmoop.com/common-core-standards/ccss-hs-a-rei-2.html (Shmoop standard page)
- http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_2.pdf (Reasoning with Equations – Worksheet 2)

A-REI.4 Basic

- http://www.illustrativemathematics.org/illustrations/618 (Two Squares are Equal)
- http://www.shmoop.com/common-core-standards/ccss-hs-a-rei-4.html (Shmoop standard page)
- http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_4.pdf (Reasoning with Equations – Worksheet 4)
- http://www.shmoop.com/common-core-standards/handouts/a-rei_worksheet_4_ans.pdf (Reasoning with Equations – Worksheet 4 Answers)

INSTRUCTIONAL STRATEGIES

OR

$$\begin{aligned} 3n + 2 &= n - 10 \\ +\quad 10 &= \quad +10 \\ \hline 3n + 12 &= n \\ -3n \quad &= -3n \\ \hline 12 &= -2n \\ n &= -6 \end{aligned}$$

OR

$$\begin{aligned} 3n + 2 &= n - 10 \\ -n \quad &= -n \\ \hline 2n + 2 &= -10 \\ -2 &= -2 \\ \hline 2n &= -12 \\ n &= -6 \end{aligned}$$

- *Connect the idea of adding two equations together as a means of justifying steps of solving a simple equation to the process of solving a system of equations. A system consisting of two linear functions such as 2x +3y = 8 and x - 3y = 1 can be solved by adding the equations together, and can be justified by exactly the same reason that solving the equation 2x - 4 = 5 can begin by adding the equation 4 = 4.* (ODE)

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
ALGEBRA **Reasoning with Equations and Inequalities (A-REI)** Represent and solve equations and inequalities graphically. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others **4.** Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**M**	**Students** A-REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately. **Major content** ▪ For example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★ **Essential questions** • Why are the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect equal to the solutions of the equation f(x) = g(x)? • Why does graphing or using a table give approximate solutions? • In what situations would you want an exact solution rather than an approximate solution or vice versa? **Essential knowledge and skills** • Solving a system of equations algebraically yields an exact solution; solving by graphing or by comparing tables of values yields an approximate solution. • The x-coordinates of the points where the graphs of the equations *y = f(x) and y = g(x)* intersect are the solutions of the equation *f(x) = g(x).* **Teaching Examples** Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions. (Does not include logarithmic functions) • Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions. **Mathematical Practices** • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *Combine polynomial, rational, radical, absolute value, and exponential functions* • *Beginning with simple, real-world examples, help students to recognize a graph as a set of solutions to an equation. For example, if the equation y = 6x + 5 represents the amount of money paid to a babysitter (i.e., $5 for gas to drive to the job and $6/hour to do the work), then every point on the line represents an amount of money paid, given the amount of time worked.* • *Explore visual ways to solve an equation such as 2x + 3 = x – 7 by graphing the functions y = 2x + 3 and y = x – 7. Students should recognize that the intersection point of the lines is at (-10, -17). They should be able to verbalize that the intersection point means that when x = -10 is substituted into both sides of the equation, each side simplifies to a value of -17. Therefore, -10 is the solution to the equation. This same approach can be used whether the functions in the original equation are linear, nonlinear or both.* • *Using technology, have students graph a function*	RESOURCE NOTES See resources in the introduction • Examples of real-world situations that involve linear functions and two-variable linear inequalities • Graphing calculators or computer software that generate graphs and tables for solving equations	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

This curriculum was developed based on the Common Core State Standards utilizing examples and strategies from various websites including Tucson, Arizona, Ohio, and New Jersey.

ALGEBRA 2 CURRICULUM Grades 10-12

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CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• Given the following equations, determine the x value that results in an equal output for both functions. $f(x)=3x-2$ $g(x)=(x+3)^2-1$ Graph the following system and give the solutions for *f(x) = g(x)*. $f(x)=\|x+2\|$ $g(x)=-\frac{1}{3}x+\frac{2}{3}$ • Graph the following system and approximate the solutions for *f(x) = g(x)*. $f(x)=\frac{x+4}{2-x}$ $g(x)=x^3-6x^2+3x+10$ (TUSD) **Academic vocabulary** • Complex roots • Conjugate pairs • Discriminant • Exponential • Extraneous solution • General form • Linear • Polynomial • Radical equation • Standard form • Zero product property **Common Student Misconceptions** • Students do not check for extraneous solutions. Although students may solve an equation correctly, they don't check for the validity of the solution. • Students do not always recognize when a quadratic equation yields complex solutions. Students simply write that the equation has no real solutions, as they learned in Algebra I. • Students do not understand what a solution to a system represents. The solution represents the point(s) of intersection of the graphs of the equations. • Students do not understand when to give an exact solution and when it is appropriate to give an approximation. The situation of a real-world problem may lead to the need to approximate a solution. • Students commonly believe that decimal representations of numbers are always exact. For example, students will write 3.14 instead of π, 1.41 instead of $\sqrt{2}$, or 2.33 instead of $2\frac{1}{3}$, and will believe that their answer is still exact. This misconception comes from evaluating expressions on a calculator and simply truncating the resulting decimal. (TUSD) ASSESSMENT PROBLEMS A-REI.11 Basic • http://www.illustrativemathematics.org/illustrations/618 A-REI.B.4, A-REI.D.11 • http://www.illustrativemathematics.org/illustrations/645 F-LE.2, F-LE.3, A-REI.11 • http://www.shmoop.com/common-core-standards/ccss-hs-a-rei-11.html (Shmoop REI.11 quiz)	*and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation.* • *Begin with simple linear equations and how to solve them using the graphs and tables on a graphing calculator. Then, advance students to nonlinear situations so they can see that even complex equations that might involve quadratics, absolute value, or rational functions can be solved fairly easily using this same strategy. While a standard graphing calculator does not simply solve an equation for the user, it can be used as a tool to approximate solutions.* • *Use the table function on a graphing calculator to solve equations. For example, to solve the equation $x^2 = x + 12$, students can examine the equations $y = x^2$ and $y = x + 12$ and determine that they intersect when $x = 4$ and when $x = -3$ by examining the table to find where the y-values are the same.* (ODE)		

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CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• http://www.illustrativemathematics.org/illustrations/618 (Two Squares are Equal) • http://www.illustrativemathematics.org/illustrations/645 (Population and Food Supply) F-LE.2, F-LE.3			
FUNCTIONS **Interpreting functions (F-IF)** Interpret functions that arise in applications in terms of the context. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	M	**Students** F F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include:* **Major content** ○ *intercepts* ○ *intervals where the function is increasing, decreasing, positive, or negative* ○ *relative maximums and minimums* ○ *symmetries* ○ *end behavior* ○ *periodicity* ★ **Essential questions** • *How can you describe the shape of a graph?* • *How can you relate the shape of a graph to the meaning of the relationship it represents?* **Essential knowledge and skills** • Key features of a graph or table may include intercepts; intervals in which the function is increasing, decreasing or constant; intervals in which the function is positive, negative or zero; symmetry; maxima; minima; and end behavior. • Given a verbal description of a relationship that can be modeled by a function, a table or graph can be constructed and used to interpret key features of that function. • Graphs can be described in terms of their relative maxima and minima; symmetries; end behavior; and periodicity. **Teaching Examples** • Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. Examples: • A rocket is launched from 180 feet above the ground at time t = 0. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet. **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning	TEACHER NOTES See instructional strategies in the introduction • *Flexibly move from examining a graph and describing its characteristics (e.g., intercepts, relative maximums, etc.) to using a set of given characteristics to sketch the graph of a function.* • *Examine a table of related quantities and identify features in the table, such as intervals on which the function increases, decreases, or exhibits periodic behavior.* • *Recognize appropriate domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number, such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers.* • *Given a table of values, such as the height of a plant over time, students can estimate the rate of plant growth. Also, if the relationship between time and height is expressed as a linear equation, students should explain the meaning of the slope of the line. Finally, if*	RESOURCE NOTES See resources in the introduction • Tables, graphs, and equations of real-world functional relationships. • Graphing calculators to generate graphical, tabular, and symbolic representations of the same function for comparison.	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

This curriculum was developed based on the Common Core State Standards utilizing examples and strategies from various websites including Tucson, Arizona, Ohio, and New Jersey.

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		1. What is a reasonable domain restriction for t in this context? 2. Determine the height of the rocket two seconds after it was launched. 3. Determine the maximum height obtained by the rocket. 4. Determine the time when the rocket is 100 feet above the ground. 5. Determine the time at which the rocket hits the ground. 6. How would you refine your answer to the first question based on your response to the second and fifth questions? • Compare the graphs of y = $3x^2$ and y = $3x^3$. • Let $R(x)=\frac{2}{\sqrt{x-2}}$. Find the domain of *R(x)*. Also find the range, zeros, and asymptotes of *R(x)*. • Let $f(x)=5x^3-x^2-5x+1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease. • It started raining lightly at 5 a.m., then the rainfall became heavier at 7a.m. By 10 a.m. the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday. (TUSD) F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ▪ *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ **Essential questions** • *How would you determine the appropriate domain for a function describing a real-world situation?* **Essential knowledge and skills** • The intervals over which a function is increasing, decreasing or constant, positive, negative or zero are subsets of the function's domain. • Determine the appropriate domain for a function describing a real-world situation. **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools	*the relationship is illustrated as a linear or non-linear graph, the student should select points on the graph and use them to estimate the growth rate over a given interval.* (ODE) • *Emphasize selection of appropriate materials*		

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		Teaching Examples • Students may explain orally, or in written format, the existing relationships. Examples: • If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. • A hotel has 10 stories above ground and 2 levels in its parking garage below ground. What is an appropriate domain for a function *T(n)* that gives the average number of times an elevator in the hotel stops at the n^{th} floor each day? strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning			
	M	F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ Major content **Essential questions** • *Given a function that describes a real-world situation, what can the average rate of change of the function tell you?* • *How do the parts of a graph of a function relate to its real-world context?* **Essential knowledge and skills** • The average rate of change of a function *y = f(x)* over an interval [a, b] is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$ **Teaching Examples** • The average rate of change of a function *y = f(x)* over an interval *[a, b]* is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$ In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (such as a falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation. **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning			

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		Examples: • Use the following table to find the average rate of change of g over the intervals [–2, –1] and [0, 2]: (table 1 below) • The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40 and 50 meter mark on a test track. ○ For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1. ○ How does the velocity of car 1 compare to that of car 2? (table 2 below) (TUSD) ASSESSMENT PROBLEMS F.IF.4 Basic • http://www.illustrativemathematics.org/illustrations/387 (rational) • http://www.illustrativemathematics.org/illustrations/649 (interpreting graphs) • http://www.illustrativemathematics.org/illustrations/637 (interpreting graphs) • http://www.illustrativemathematics.org/illustrations/1279 (quadratic) • http://www.illustrativemathematics.org/illustrations/650 (interpreting graphs) • http://www.illustrativemathematics.org/illustrations/639 (interpreting graphs) • http://www.illustrativemathematics.org/illustrations/595 (trig function) • http://www.shmoop.com/common-core-standards/ccss-hs-a-f-if-4.html F.IF.4 Advanced • http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html • http://www.illustrativemathematics.org/illustrations/386 (rational) • http://www.illustrativemathematics.org/illustrations/394 (alternate version of previous problem)			

x	$g(x)$
-2	2
-1	-1
0	-4
2	-10

	Car 1	Car 2
d	t_1	t_2
10	4.472	1.742
20	6.325	2.899
30	7.746	3.831
40	8.944	4.633
50	10	5.348

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CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• http://www.illustrativemathematics.org/illustrations/804 (logistic growth) • http://www.illustrativemathematics.org/illustrations/800 (logistic growth) F.IF.5 Basic • http://www.illustrativemathematics.org/illustrations/387 (rational) • http://www.illustrativemathematics.org/illustrations/631 (linear) • http://www.shmoop.com/common-core-standards/ccss-hs-f-if-5.html F.IF.5 Advanced • http://www.illustrativemathematics.org/illustrations/386 (tabular; rational function) • http://www.illustrativemathematics.org/illustrations/595 (trig function) F.IF.6 Basic • http://www.illustrativemathematics.org/illustrations/577 • http://www.shmoop.com/common-core-standards/ccss-hs-f-if-6.html			
FUNCTIONS **Interpreting functions (F-IF)** Analyze functions using different representations. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**S**	**Students** F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ **Supporting content** a. Graph linear and quadratic functions and show intercepts, maxima, and minima. (F.IF.7a) b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (F.IF.7b) c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (F.IF.7c) d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. (F.IF.7d) e. Graph exponential and logarithmic functions showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (F.IF.7e) **Essential questions** • *How do you determine which type of function best models a given situation?* **Essential knowledge and skills** • Key features of a graph or table may include intercepts; intervals in which the function is increasing, decreasing or constant; intervals in which the function is positive, negative or zero; symmetry; maxima; minima; end behavior; asymptotes; domain; range and periodicity. **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★	TEACHER NOTES See instructional strategies in the introduction • *Focus on using key features to guide selection of appropriate type of model function* • *Explore various families of functions and help students to make connections in terms of general features. For example, just as the function $y = (x + 3)^2 - 5$ represents a translation of the function $y = x$ by 3 units to the left and 5 units down, the same is true for the function $y = \| x + 3 \| - 5$ as a translation of the absolute value function $y = \| x \|$.* • *Discover that the factored form of a quadratic or polynomial equation can be used to determine the zeros, which in turn can be used to identify maxima, minima and end behaviors.*	RESOURCE NOTES See resources in the introduction • Graphing utilities on a calculator and/or computer can be used to demonstrate the changes in behavior of a function as various parameters are varied. • Real-world problems, such as maximizing the area of a region bound by a fixed perimeter fence, can help to illustrate applied uses of families of functions.	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

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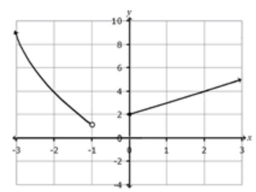
		• The graph of a trigonometric function shows period, amplitude, midline and asymptotes. • The graph of a polynomial function shows zeros and end behavior. **Teaching Examples** • In Algebra I, students looked at F-IF.7c as the relationship between zeros of quadratic functions and their factored forms. • F-IF.7e links to F-TF.2 and 5 regarding the extension of trig functions. • Logarithmic functions do not need to be addressed in Algebra II in terms of graphing. • Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators, graphing programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Describe key characteristics of the graph of f(x) = $\lvert x - 3\rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x)=\begin{cases} x+2 & \text{for} \quad x \geq 0 \\ -x^2 & \text{for} \quad x < -1 \end{cases}$ Solution:  • Graph the function *f(x) = 2x* by creating a table of values. Identify the key characteristics of the graph. • Graph *f(x) = 2 tan x – 1*. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of *f(x) = sin x* and *f(x) = cos x*. What are the similarities and differences between the two graphs? (TUSD) • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	• *Use various representations of the same function to emphasize different characteristics of that function. For example, the y-intercept of the function y = x^2 -4x – 12 is easy to recognize as (0, -12). However, rewriting the function as y = (x – 6)(x + 2) reveals zeros at (6, 0) and at (-2, 0). Furthermore, completing the square allows the equation to be written as y = $(x – 2)^2$ – 16, which shows that the vertex (and minimum point) of the parabola is at (2, -16).* • *Examine multiple real-world examples of exponential functions so that students recognize that a base between 0 and 1 (such as an equation describing depreciation of an automobile [f(x)=15,000$(0.8)^x$ represents the value of a $15,000 automobile that depreciates 20% per year over the course of x years]) results in an exponential decay, while a base greater than 1 (such as the value of an investment over time [f(x)=5,000$(1.07)^x$ represents the value of an investment of $5,000 when increasing in value by 7% per year for x years]) illustrates growth.* (ODE)		
	S	F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **Supporting content**			

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (F.IF.8a) b. Use the properties of exponents to interpret expressions for exponential functions. ○ *For example, identify percent rate of change in functions such as:* ▪ $y = (1.02)^t$ ▪ $y = (0.97)^t$ ▪ $y = (1.01)^{12t}$ ▪ $y = (1.2)^{t/10}$ *and classify them as representing exponential growth or decay.* (F.IF.8b) **Essential questions** • *How do different forms of a function help you to identify key features?* **Essential knowledge and skills** • For a function of the form $f(t) = ab^t$, if b > 1 the function represents exponential growth; if b < 1 the function represents exponential decay. **Teaching Examples** • In Algebra I, students focused on this standard with linear, exponential and quadratic functions. Example: • Write the following function in a different form and explain what each form tells you about the function: $f(x) = x^3 - 6x^2 + 3x + 10$ (TUSD) **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure			
	S	F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Supporting content** ○ *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (F.IF.9) **Essential questions** • *How can you compare properties of two functions if they are represented in different ways?* **Essential knowledge and skills** • A function can be represented algebraically, graphically, numerically in tables, or by verbal descriptions. **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools	• *Focus on using key features to guide selection of appropriate type of model function*		

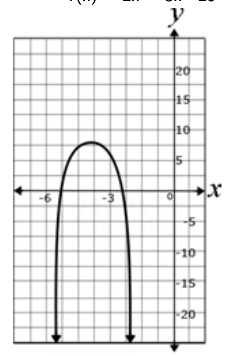
CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS

Teaching Examples

Example:

- Examine the functions below. Which function has the larger maximum? How do you know?

$f(x) = -2x^2 - 8x + 20$



(TUSD)

strategically

- Attend to precision
- Look for and make use of structure

Academic vocabulary

- Amplitude
- Asymptote
- Delta
- Dependent variable
- Domain
- Domain restriction
- End behavior
- Exponential decay
- Exponential growth
- Independent variable
- Interval
- Maxima
- Minima
- Period
- Periodic function
- Range
- Rate of change
- Subset
- Symmetry
- Zeros

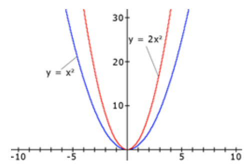
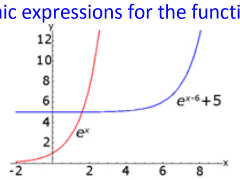
Common Student Misconceptions

- Students tend to focus on the y values of the graph instead of the x values of the interval, when identifying key features of a graph. Students have difficulty understanding domain.
- Students will place the independent variable on the y-axis. Students often create graphs that are not functions by the vertical line test because they graph the independent variable on the y-axis as opposed to the x-axis.
- Students don't move the asymptote appropriately when shifting functions. Students tend to leave the asymptote in the old place, and then draw the new function crossing the asymptote.
- Students have a hard time grasping the concept of domain. Students need assistance connecting domain to interval over which key features occur, such as where the function is increasing.
- Students have difficulty with periodic functions, such as sin(x) and cos(x), because of the wave nature of the graph. Students will need assistance making a connection between the words of the task or problem and the graph that represents it. (TUSD)

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		ASSESSMENT PROBLEMS F.IF.7 Basic • http://www.illustrativemathematics.org/illustrations/388 A-SSE.B.3, F-IF.C.7 (Graphs of Quadratic Functions) • http://www.illustrativemathematics.org/illustrations/803 (Identifying graphs of functions) • http://www.illustrativemathematics.org/illustrations/627 F-IF.C.7.c (Graphs of power functions) • http://www.illustrativemathematics.org/illustrations/803 (7e) (exponential/logistic growth) • http://www.shmoop.com/common-core-standards/ccss-hs-f-if-7.html • http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_7.pdf • http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_7_ans.pdf F.IF.7 Advanced • http://www.illustrativemathematics.org/illustrations/388 (7c) (quadratic) F.IF.8 Basic • http://www.illustrativemathematics.org/illustrations/640 F-IF.C.8.a (Which Function?) • http://www.illustrativemathematics.org/illustrations/375 F-IF.C.8.a, A-REI.B.4.b (Springboard Dive) • http://www.illustrativemathematics.org/illustrations/640 (8a) (quadratic) • http://www.illustrativemathematics.org/illustrations/375 (also A-REI.4b) • http://www.shmoop.com/common-core-standards/ccss-hs-f-if-8.html • http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_8.pdf • http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_8_ans.pdf F.IF.9 Basic • http://www.illustrativemathematics.org/illustrations/1279 F-IF.B.4, F-IF.C.9 (Throwing Baseballs) • http://www.parcconline.org/samples/mathematics/high-school-functions • http://www.illustrativemathematics.org/illustrations/1279 (quadratic) • http://www.shmoop.com/common-core-standards/ccss-hs-f-if-9.html • http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_9.pdf • http://www.shmoop.com/common-core-standards/handouts/f-if-worksheet_9_ans.pdf			
FUNCTIONS **Building Functions (F-BF)** Build a function that models a relationship between two quantities	**M**	**Students** F-BF.1 Write a function that describes a relationship between two quantities. ★ **Major content** b. Combine standard function types using arithmetic operations. ○ *For example, build a function that models the temperature of a cooling body by adding a constant function t.* (F-BF.1b) **Essential questions** **Mathematical Practices**	TEACHER NOTES See instructional strategies in the introduction *Include all types of functions studied* • *Provide a real-world example (e.g., a table*	RESOURCE NOTES See resources in the introduction • Hands-on materials (e.g., paper folding, building progressively larger shapes using pattern blocks, etc.) can	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department		INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others **4.** Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning		• *What data would you need to write a function to model a given situation?* • *How do you translate a description of the relationship between* **<u>Essential knowledge and skills</u>** • A function is a relationship between two quantities. • The function representing a given situation may be a combination of more than one standard function. • Standard functions may be combined through arithmetic operations. **<u>Teaching Examples</u>** Examples: • You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • You are making an open box out of a rectangular piece of cardboard with dimensions 40 cm by 30 cm by cutting equal squares out of the four corners and then folding up the sides. How big should the squares be to maximize the volume of the box? Draw a diagram to represent the problem and write an appropriate equation to solve. • Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (TUSD) **<u>Academic vocabulary</u>** • Contract • Expand • Inverse function • Inverse operation • Odd/even function • Parameters • Reflection • Shrink • Standard function • Stretch • Symmetrical • Transformation • Translation/Shift	• Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	*showing how far a car has driven after a given number of minutes, traveling at a uniform speed), and examine the table by looking "down" the table to describe a recursive relationship, as well as "across" the table to determine an explicit formula to find the distance traveled if the number of minutes is known.* • *Write out terms in a table in an expanded form to help students see what is happening. For example, if the y-values are 2, 4, 8, 16, they could be written as 2, 2(2), 2(2)(2), 2(2)(2)(2), etc., so that students recognize that 2 is being used multiple times as a factor.* • *Focus on one representation and its related language – recursive or explicit – at a time so that students are not confusing the formats.* • *Provide examples of when functions can be combined, such as determining a function describing the monthly cost for owning two vehicles when a function for the cost of each (given the number of miles driven) is known.* • *Using visual approaches (e.g., folding a piece of paper in half multiple times), use the visual models to generate sequences of numbers that can be explored and described with both recursive and explicit formulas. Emphasize that there are times when one form to describe the*	be used as a visual source to build numerical tables for examination.	<u>FORMATIVE/ SUMMATIVE ASSESSMENTS</u> See assessments in the introduction

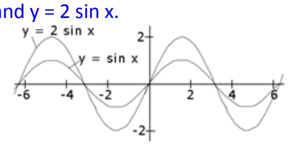
CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		ASSESSMENT PROBLEMS F.BF.1 Basic • http://www.illustrativemathematics.org/illustrations/230 • http://www.illustrativemathematics.org/illustrations/241 (linear) • http://www.illustrativemathematics.org/illustrations/533 (exponential) • http://www.illustrativemathematics.org/illustrations/386 (rational) F.BF.1 Advanced • http://www.illustrativemathematics.org/illustrations/75 (quadratic) • http://www.illustrativemathematics.org/illustrations/72 (rational) • http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html	*function is preferred over the other.* (ODE)		
FUNCTIONS **Building Functions (F-BF)** Build new functions from existing functions Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**A**	**Students** F-BF.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. **Additional content** Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **Essential questions** • *Create a graph and explain what transformation(s) were done on the parent function to create that graph.* • *What are the transformations that can be done to a graph and how can they be represented algebraically?* • *How do you determine if a graph is odd, even or neither?* • *Why are the two descriptions of an even function equivalent?* • *Why are the two descriptions of an odd function equivalent?* **Essential knowledge and skills** • *f(x) + k* will translate the graph of the function f(x) up or down by k units. • *k•f(x)* will expand or contract the graph of the *function f(x)* vertically by a factor of k. If *k* < 0 the graph will reflect across the x-axis. • *f(kx)* will expand or contract the graph of the function f(x) horizontally by a factor of *k*. If *k* < 0 the graph will reflect across the y-axis. • *f(x + k*) will translate the graph of the function f(x) left or right by k units. **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *Include simple radical, rational, and exponential functions: emphasize common effect of each transformation across function types* • *Use graphing calculators or computers to explore the effects of a constant in the graph of a function. For example, students should be able to distinguish between the graphs of* $y = x^2$, $y = 2x^2$, $y = x^2 + 2$, $y = (2x)^2$, *and* $y = (x + 2)^2$. *This can be accomplished by allowing students to work with a single parent function and examine numerous parameter changes to make generalizations.* • *Distinguish between even and odd functions by providing several examples and helping students to recognize that a function is even if f(-x) = f(x) and is odd if f(-x) = -f(x). Visual approaches to identifying*	RESOURCE NOTES See resources in the introduction • Graphing calculator that can be used to explore the effects of parameter changes on a graph	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		• If *f(–x) = f(x)* then the function is even, therefore its graph is symmetrical across the y-axis. • If *f(–x) = –f(x)* then the function is odd, therefore its graph is symmetrical across the origin. **Teaching Examples** Examples: • Explore the functions *f(x) = 3x, g(x)= 5x*, and $h(x)=\frac{1}{2}x$ with a calculator to develop a relationship between the coefficient on x and the slope of a line. • Compare the graphs of *f(x) = 3x* with those of *g(x)* = 3x + 2 and *h(x)* = 3x – 1 to see that parallel lines have the same slope AND to explore the effect of the transformations of the function *f(x) = 3x*, such that *g(x) = f(x)+2 and h(x) = f(x) – 1.* • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions. •  • Describe the effect of varying the parameters a, h, and k on the shape and position of the graph of $f(x)=a(x-h)^2+k$. • Compare the shape and position of the graphs of $f(x)=e^x$ and $g(x)=e^{x-6}+5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions.  • Describe the effect of varying the parameters a, h, and k on the shape and position of the graph $f(x)=ab^{(x-h)}+k$, orally or in written format. What effect do values between 0 and 1 have? What effect	*the graphs of even and odd functions can be used as well.* • *Provide examples of inverses that are not purely mathematical to introduce the idea. For example, given a function that names the capital of a state, f(Ohio) = Columbus. The inverse would be to input the capital city and have the state be the output, such that* f^{-1}*(Denver) = Colorado.* • *Some information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics and goes beyond the mathematics that all students should study in order to be college- and career-ready:* • *Students should also recognize that not all functions have inverses. Again using a nonmathematical example, a function could assign a continent to a given country's input, such as g(Singapore) = Asia. However,* g^{-1}*(Asia) does not have to be Singapore (e.g., it could be China).* • *Exchange the x and y values in a symbolic functional equation and solve for y to determine the inverse function. Recognize that putting the output from the original function into the input of the inverse results in the original input value.*		

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS

do negative values have?

- Compare the shape and position of the graphs of y = sin x and y = 2 sin x.



(TUSD)

A

F-BF.4 Find inverse functions. **Additional content**

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.

- *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.* (F-BF.4a)

Essential questions

- *How do you determine if two functions are inverses of one another?*
- *Given a function, how do you find its inverse?*
- *How do you determine whether a function has an inverse?*

Essential knowledge and skills

- Two functions f and g are inverses of one another if for all values of x in the domain of f, *f(x)=y* and *g(y)=x*.
- Not all functions have an inverse.

Teaching Examples

Examples:

- For the function $h(x) = (x-2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. Graph *h(x)* and $h^{-1}(x)$ and explain how they relate to each other graphically.
- Find a domain for $f(x) = 3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.
- Find the inverse of the function $f(x) = \frac{3x+2}{2x-1}$, if it exists, or explain why the inverse doesn't exist. Describe the domain and range of *f(x)* and its inverse (if it exists). (TUSD)

Mathematical Practices

- *Reason abstractly and quantitatively*
- *Model with mathematics* ★
- *Use appropriate tools strategically*
- *Attend to precision*
- *Look for and make use of structure*

Instructional Strategies

- *Also, students need to recognize that exponential and logarithmic functions are inverses of one another and use these functions to solve real-world problems.*
- *Nonmathematical examples of functions and their inverses can help students to understand the concept of an inverse and determining whether a function is invertible.*
- *Include simple radical, rational, and exponential functions: emphasize common effect of each transformation across function types* (ODE)

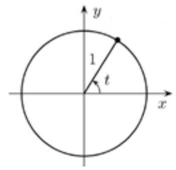
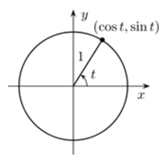
CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		Academic vocabulary • Contract • Expand • Inverse function • Inverse operation • Odd/even function • Parameters • Reflection • Shrink • Standard function • Stretch • Symmetrical • Transformation • Translation/Shift **Common Student Misconceptions** • Students often confuse the shift of a function with the stretch of a function. For example, students think k•f(x) represents a shift (translation) rather than a stretch. • Students do not understand the need for restricted domains when finding the inverses of functions. For example, $f(x) = x^2$ (domain all real numbers) has no inverse, but $f(x) = x^2$ restricted to $x \geq 0$ does have an inverse. • Students often mistake the direction of a horizontal shift. For example, students will interpret $3\sin(x-\frac{\pi}{2})+4$ as a shift to the left by $\frac{\pi}{2}$ units, rather than a shift to the right. (TUSD) ASSESSMENT PROBLEMS F.BF.3 Basic • http://www.illustrativemathematics.org/illustrations/232 • http://www.illustrativemathematics.org/illustrations/741 • http://www.illustrativemathematics.org/illustrations/742 • http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html F.BF.3 Advanced • http://www.illustrativemathematics.org/illustrations/695 (quadratic) • http://www.illustrativemathematics.org/illustrations/505 (quadratic) F.BF.4 Basic • http://www.illustrativemathematics.org/illustrations/501 (linear) • http://www.shmoop.com/common-core-standards/ccss-hs-bf-4.html			
FUNCTIONS **Linear, Quadratic, and Exponential Models★ (F-LE)** Construct and compare linear, quadratic, and exponential models and solve problems Use **Mathematical Practices** to	**S**	**Students** F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★**Supporting content** **Essential questions** • How do logarithms help you to solve exponential functions? • How do you evaluate a logarithm using technology? **Essential knowledge and skills** • The solution to an exponential function can be found using logarithms. **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically	TEACHER NOTES See instructional strategies in the introduction • *Logarithms as solutions for exponentials* • *Compare tabular representations of a variety of functions to show that linear functions have a first*	RESOURCE NOTES See resources in the introduction • Examples of real-world situations that apply linear and exponential functions to compare their behaviors • Graphing calculators or computer software that generate graphs and	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others **4.** Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning		**<u>Teaching Examples</u>** • Solve $200\,e^{0.04t} = 450$ for *t*. Solution: • We first isolate the exponential part by dividing both sides of the equation by 200. ○ $e^{0.04t} = 2.25$ • Now we take the natural logarithm of both sides. ○ $\ln(e^{0.04t}) = \ln 2.25$ • The left hand side simplifies to 0.04*t*. ○ $0.04t = \ln 2.25$ • Lastly, divide both sides by 0.04. ○ $t = \ln(2.25) / 0.04$ ○ $t \approx 20.3$ (TUSD) • Attend to precision • Look for and make use of structure **<u>Academic vocabulary</u>** • Base • Common logarithm • Exponential function • Logarithm • Natural Logarithm **<u>Common Student Misconceptions</u>** • Students do not understand what the base of a logarithm means. For example, students often use base 10 logarithms to solve exponential equations with other bases. They overgeneralize being able to use the log button on their calculators. • Students commonly make mistakes in the order of operations while solving equations. For example, to solve $4e^x = 1$, students will write $\ln(4e^x) = \ln 1$ and then either get stuck or else write $4x = 0$, so $x = 0$. The correct solution is to divide both sides by 4 first and then take the natural log, leaving $\ln(e^x) = \frac{1}{4}$, so that $x = \ln\frac{1}{4}$. (TUSD) ASSESSMENT PROBLEMS F.LE.4 Basic • http://www.illustrativemathematics.org/illustrations/370 • http://www.illustrativemathematics.org/illustrations/570 • http://www.illustrativemathematics.org/illustrations/369 • http://www.illustrativemathematics.org/illustrations/760 • http://www.illustrativemathematics.org/illustrations/214 • http://www.illustrativemathematics.org/illustrations/382 • http://www.shmoop.com/common-core-standards/ccss-hs-le-4.html F.LE.4 Advanced • http://www.illustrativemathematics.org/illustrations/638 • http://www.illustrativemathematics.org/illustrations/784	*common difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal x-intervals).* • *Apply linear and exponential functions to real-world situations. For example, a person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.* • *Provide examples of arithmetic and geometric sequences in graphic, verbal, or tabular forms, and have students generate formulas and equations that describe the patterns.* • *Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the y (output) values of the exponential function eventually exceed those of polynomial functions.* • *Have students draw the graphs of exponential and other polynomial functions on a graphing calculator or computer utility and examine the fact that the exponential curve will eventually get higher than the polynomial function's graph. A simple example would be to compare the*	tables of functions. A graphing tool such as the one found at nlvm.usu.edu is one option.	See assessments in the introduction

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

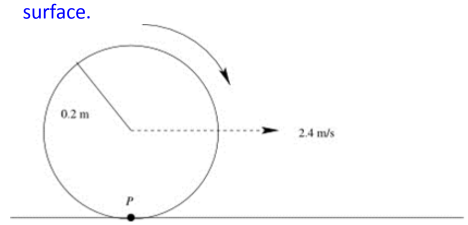
CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
			graphs (and tables) of the functions $y=x^2$ and $y=2^x$ to find that the y values are greater for the exponential function when x > 4. • *Help students to see that solving an equation such as $2^x = 300$ can be accomplished by entering $y = 2^x$ and y = 300 into a graphing calculator and finding where the graphs intersect, by viewing the table to see where the function values are about the same, as well as by applying a logarithmic function to both sides of the equation.* • *Use technology to solve exponential equations such as $3●10^x = 450$. (In this case, students can determine the approximate power of 10 that would generate a value of 150.) Students can also take the logarithm of both sides of the equation to solve for the variable, making use of the inverse operation to solve.* (ODE)		
FUNCTIONS **Trigonometric Functions** Extend the domain of trigonometric functions using the unit circle. **(F-TF)** Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and	**A**	**Students** F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **Additional content** **Essential questions** • *Explain what a radian measure is.* **Essential knowledge and skills** • The unit circle is a circle with radius of length 1 centered at the origin. • The radian measure of an angle is the length of the arc on the unit circle subtended by the angle. **Teaching Examples** • What is the radian measure of the angle t in the **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision	TEACHER NOTES See instructional strategies in the introduction • *Use a compass and straightedge to explore a unit circle with a fixed radius of 1. Help students to recognize that the circumference of the circle is 2π, which represents the number of radians for one complete revolution around the circle. Students can*	RESOURCE NOTES See resources in the introduction • Compass and straightedge to explore the unit circle and to draw sine and cosine curves and describe their periodicity. • Graphing calculators or computer graphing tools to determine	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
quantitatively 3. Construct viable arguments and critique the reasoning of others **4.** Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	 **A**	diagram below? • Look for and make use of structure  (TUSD) F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **Additional content** **Essential questions** • *What is the unit circle, and why do you need it?* • *How does the unit circle let you extend trigonometric functions to all real numbers?* **Essential knowledge and skills** • Angles on the unit circle are measured counterclockwise from the point (1, 0). • Trigonometric functions can be extended to the domain of all real numbers using the unit circle. **Teaching Examples** • The coordinates (x, y) of any point on the unit circle are given by x = cos t, y = sin t, where t is the radian measure of the angle from the positive x-axis.  (TUSD) **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure ASSESSMENT PROBLEMS F-LE.2 Basic • http://www.illustrativemathematics.org/illustrations/645 (Population and Food Supply) F-LE.3 • http://www.shmoop.com/common-core-standards/ccss-hs-f-tf-1.html • http://www.shmoop.com/common-core-standards/ccss-hs-f-tf-2.html	*determine that, for example, π/4 radians would represent a revolution of 1/8 of the circle or 45°.* • *Having a circle of radius 1, the cosine, for example, is simply the x-value for any ordered pair on the circle (adjacent/hypotenuse where adjacent is the x-length and hypotenuse is 1). Students can examine how a counterclockwise rotation determines a coordinate of a particular point in the unit circle from which sine, cosine, and tangent can be determined.* • *Some information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics and goes beyond the mathematics that all students should study in order to be college- and career-ready:* • *Some students can use what they know about 30-60-90 triangles and right isosceles triangles to determine the values for sine, cosine, and tangent for π/3, π/4, and π/6. In turn, they can determine the relationships between, for example, the sine of π/6, 7π/6, and 11π/6, as all of these use the same reference angle and knowledge of a 30-60-90 triangle.* • *Provide students with real-world examples of periodic functions. One good*	radian measures and to find values of the sine, cosine, and tangent functions for any given x input value.	See assessments in the introduction

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
			example is the average high (or low) temperature in a city in Rhode Island for each of the 12 months. These values are easily located at weather.com and can be graphed to show a periodic change that provides a context for exploration of these functions.. (ODE) •		
FUNCTIONS **Trigonometric Functions (F-TF)** Model periodic phenomena with trigonometric functions. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others **4.** Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	**A**	**Students** F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★ **Additional content** **Essential questions** • *What are the key features of a trigonometric function?* • *What kinds of phenomena can be modeled by trigonometric functions? Give and example.* • *What information about a situation do you need in order to model it with a trigonometric function?* • *What do the amplitude, frequency, and midline of a trigonometric function tell you about the situation it models?* • *How are period and frequency related?* **Essential knowledge and skills** • Trigonometric functions can be used to model periodic phenomena. • In order to model a periodic phenomenon, you need to know the amplitude, frequency or period, and midline. **Teaching Examples** Example: • The temperature of a chemical reaction oscillates between a low of 20°C and a high of 120°C. The temperature is at its lowest point when $t = 0$ and completes one cycle over a six-hour period. a. Sketch the temperature, T, against the elapsed time, t, over a 12-hour period. b. Find the period, amplitude, and the midline of the graph you drew in part (1). c. Write a function to represent the relationship between time and temperature. **Mathematical Practices** • Reason abstractly and quantitatively • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *Allow students to explore real-world examples of periodic functions. Examples include average high (or low) temperatures throughout the year, the height of ocean tides as they advance and recede, and the fractional part of the moon that one can see on each day of the month. Graphing some real-world examples can allow students to express the amplitude, frequency, and midline of each.* • *Help students to understand what the value of the sine (cosine, or tangent) means (e.g., that the number represents the ratio of two sides of a right triangle having that angle measure).* • *Using graphing calculators or computer software, as well as graphing simple examples by hand, have students graph a variety of trigonometric functions in which the amplitude,*	RESOURCE NOTES See resources in the introduction • A list of real-world applications of periodic situations that can be modeled by using trigonometric functions for students to explore. • Graphing calculators or computer software to generate the graphs of trigonometric functions.	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

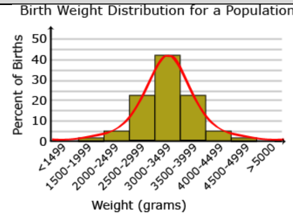
CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		d. What will the temperature of the reaction be 14 hours after it began? e. At what point(s) during a 24-hour day will the reaction have a temperature of 60°C? • A wheel of radius 0.2 meters begins to move along a flat surface so that the center of the wheel moves forward at a constant speed of 2.4 meters per second. At the moment the wheel begins to turn, a marked point P on the wheel is touching the flat surface.  Write an algebraic expression for the function y that gives the height (in meters) of the point P, measured from the flat surface, as a function of *t*, the number of seconds after the wheel begins moving. From http://illustrativemathematics.org (TUSD) ASSESSMENT PROBLEMS F.TF.5 Basic • http://www.illustrativemathematics.org/illustrations/816 (Foxes or Rabbits 2) • http://www.illustrativemathematics.org/illustrations/817 (Foxes or Rabbits 3) • http://www.illustrativemathematics.org/illustrations/595 F-TF.B.5, F-IF.B.4 (As the Wheels Turn) • http://www.shmoop.com/common-core-standards/ccss-hs-f-tf-5.html	*frequency, and/or midline is changed. Students should be able to generalize about parameter changes, such as what happens to the graph of y = cos(x) when the equation is changed to y = 3cos(x) + 5.* • *Some information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics and goes beyond the mathematics that all students should study in order to be college- and career-ready:* • *Some students can explore the inverse trigonometric functions, recognizing that the periodic nature of the functions depends on restricting the domain. These inverse functions can then be used to solve real-world problems involving trigonometry with the assistance of technology.* (ODE)		
FUNCTIONS **Trigonometric Functions (F-TF)** Prove and apply trigonometric identities. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively	**A**	**Students** F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. **Additional content** **Essential questions** • How can you prove the Pythagorean identity? • How can you find sin(θ), cos(θ), or tan(θ) using the Pythagorean identity? **Essential knowledge and skills** • The Pythagorean identity states that $\sin^2(\theta)$ + **Mathematical Practices** • Reason abstractly and quantitatively • Construct viable arguments and	TEACHER NOTES See instructional strategies in the introduction • *In the unit circle, the cosine is the x-value, while the sine is the y-value. Since the hypotenuse is always 1, the Pythagorean relationship $\sin^2(\vartheta)+\cos^2(\vartheta)=1$ is always*	RESOURCE NOTES See resources in the introduction • Drawings of the unit circle can be useful in showing why the Pythagorean relationship must be true.	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
3. Construct viable arguments and critique the reasoning of others **4.** Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning		$\cos^2(\theta) = 1$. • The Pythagorean identity can be used to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given one of those quantities and the quadrant of the angle. **Teaching Examples** • Prove the Pythagorean identity. • Given that $\cos\theta = \frac{\sqrt{3}}{2}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find the values of $\sin(\theta)$ and $\tan(\theta)$. (TUSD) critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure **Academic vocabulary** • Amplitude • Cosine • Frequency • Midline • Oscillation • Period • Periodic function • Pythagorean Identity • Radian measure • Sine • Tangent • Trigonometric function • Unit circle **Common Student Misconceptions** • Students commonly do not remember the trigonometry relations that they learned from special right triangles correctly. For example, they will say that $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$ rather than $\frac{\sqrt{3}}{2}$. • Students often confuse the sine and the cosine of an angle. For example, students will say that $\cos\left(\frac{\pi}{6}\right) = \frac{1}{2}$ and $\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, rather than the other way around. • Students frequently evaluate trigonometric functions with degree measures instead of radian measures. They forget, or do not understand, that they must either convert from degrees to radians before evaluating the function or set their calculators to work with degrees. • Students are confused about the connection between the coordinates of points on the unit circle and the graphs of y = cos x and y = sin x. • Students have trouble determining the quadrant of an angle from the signs of the sine and cosine, and vice versa. (TUSD) ASSESSMENT PROBLEMS • http://www.shmoop.com/common-core-standards/ccss-hs-f-tf-8.html	*true. Students can make a connection between the Pythagorean Theorem in geometry and the study of trigonometry by proving this relationship. In turn, the relationship can be used to find the cosine when the sine is known, and vice-versa. Provide a context in which students can practice and apply skills of simplifying radicals.* • *Some information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics and goes beyond the mathematics that all students should study in order to be college- and career-ready:* • *Some students can explore other trigonometric identities, such as the half-angle, double-angle, and addition/subtraction formulas to extend on the Pythagorean relationship. Formulas should be proven and then used to determine exact values when given an angle measure, to prove identities, and to solve trigonometric equations. For example, by dividing the formula $\sin^2(\vartheta)+\cos^2(\vartheta)=1$ by $\cos^2(\vartheta)$, a new formula is generated ($\tan^2(\vartheta)+1=\sec^2(\vartheta)$).* (ODE)		ASSESSMENTS See assessments in the introduction.

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
STATISTICS AND PROBABILITY **Interpreting Categorical and Quantitative Data (S-ID)** Summarize, represent, and interpret data on a single count or measurement variable. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning		**Students** S-ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **Essential questions** • *What can a normal distribution tell you about a data set?* • *When can a data set be fitted with a normal distribution?* • *Why can't a normal distribution be used to describe all data sets?* • *How can you estimate population percentages from a data set?* **Essential knowledge and skills** • A normal distribution can describe some, but not all, data sets. • Each normal distribution has a well-defined mean and standard deviation. • The mean and standard deviation of a data set can be used to find the best-fit normal distribution for that data set. • The normal distribution of a set of population data can be used to estimate population percentages. **Teaching Examples** Examples: • Determine which situation(s) is/are best modeled by a normal distribution. Explain your reasoning. ○ Annual income of a household in the U.S. ○ Weight of babies born in one year in the U.S. • The bar graph below gives the birth weight of a population of 100 chimpanzees. The line shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000-3999 grams. **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *It is helpful for students to understand that a statistical process is a problem-solving process consisting of four steps: formulating a question that can be answered by data; designing and implementing a plan that collects appropriate data; analyzing the data by graphical and/or numerical methods; and interpreting the analysis in the context of the original question. Opportunities should be provided for students to work through the statistical process. In Grades 6-8, learning has focused on parts of this process. Now is a good time to investigate a problem of interest to the students and follow it through. The richer the question formulated, the more interesting is the process. Teachers and students should make extensive use of resources to perfect this very important first step. Global web resources can inspire projects.* • *Although this domain addresses both categorical and quantitative data, there is no reference in the Standards 1 - 4 to categorical data. Note that Standard 5 in the next cluster (Summarize, represent, and interpret*	RESOURCE NOTES See resources in the introduction	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		 (TUSD) **Academic vocabulary** • Bell curve • Mean • Median • Mode • Normal distribution • Sigma • Standard deviation • Variance **Common Student Misconceptions** • Students confuse the normal distribution with the uniform distribution. For example, students often believe that any random number always comes from a uniform distribution; they may not realize that the data described by a normal distribution is also random. • Students often misunderstand how to compute probabilities from a normal distribution. For example, students think that the probability of a particular x-value occurring is given by the height of the bell curve at that point. However, the normal distribution is used to calculate the probability that a data point will be within a given interval by finding the area under the curve within that interval. • Students often confuse the variance and the standard deviation. Students forget that the standard deviation of a distribution is the square root of the variance. (TUSD) ASSESSMENT PROBLEMS S-ID.4 Basic • http://www.illustrativemathematics.org/illustrations/216 • http://www.illustrativemathematics.org/illustrations/1020 • http://www.shmoop.com/common-core-standards/ccss-hs-s-id-4.html S-ID.4 Advanced • http://www.illustrativemathematics.org/illustrations/1218	*data on two categorical and quantitative variables) addresses analysis for two categorical variables on the same subject. To prepare for interpreting two categorical variables in Standard 5, this would be a good place to discuss graphs for one categorical variable (bar graph, pie graph) and measure of center (mode).* • *Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's pizza preferences) or measurement on the vertical (radish root growth over time - days).* • *Measures of center and spread for data sets without outliers are the mean and standard deviation, whereas median and interquartile range are better measures for data sets with outliers.* • *Introduce the formula of standard deviation by reviewing the previously learned MAD (mean absolute deviation). The MAD is very intuitive and gives a solid foundation for developing the more complicated standard deviation measure.* (ODE)		

ALGEBRA 2 CURRICULUM Grades 10-12

Curriculum Writers: Robin Broman and Thomas Yeaw

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
STATISTICS AND PROBABILITY **Making Inferences and Justifying Conclusions (S-IC)** Understand and evaluate random processes underlying statistical experiments. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	S S	**Students** S-IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **Supporting content** __Essential questions__ • How can you determine if a model is consistent with the results of a simulation or experiment? __Essential knowledge and skills__ • If a model is appropriate for a given situation, the experimental probability of an event will approach the theoretical probability as the sample size increases. __Teaching Examples__ Example: • Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students. 1. Describe the parameter of interest and a statistic the students could use to estimate the parameter. 2. Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning. 3. The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness. 4. Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above. (TUSD) From: illustrativemathematics.org __Mathematical Practices__ • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure S-IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. **Supporting content** ▪ For example, *a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	TEACHER NOTES See instructional strategies in the introduction • *Inferential statistics based on Normal probability models is a topic for Advanced Placement Statistics (e.g., t-tests). The idea here is that all students understand that statistical decisions are made about populations (parameters in particular) based on a random sample taken from the population and the observed value of a sample statistic (note that both words start with the letter "s"). A population parameter (note that both words start with the letter "p") is a measure of some characteristic in the population such as the population proportion of American voters who are in favor of some issue, or the population mean time it takes an Alka Seltzer tablet to dissolve.* • *As the statistical process is being mastered by students, it is instructive for them to investigate questions such as "If a coin spun five times produces five tails in a row, could one conclude that the coin is biased toward tails?" One way a student might answer this is by building a model of 100 trials by experimentation or simulation of the number of times a truly fair coin produces five tails in a row*	RESOURCE NOTES See resources in the introduction • TI-83/84 and TI emulator	ASSESSMENT NOTES __REQUIRED COMMON ASSESSMENTS__ • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS __SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS__ See assessments in the introduction

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		. **<u>Essential questions</u>** • *What is the difference between an experimental probability and a theoretical probability?* **<u>Essential knowledge and skills</u>** • Experiments must be repeated to verify a model. • Large numbers of trials can be performed using computer simulations. **<u>Teaching Examples</u>** For S-IC.2, include comparing theoretical and empirical results to evaluate the effectiveness of a treatment. • Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using computer random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials. • The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model-building verification process. Examples: • Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability? • A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (TUSD) **<u>Mathematical Practices</u>** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure **<u>Academic vocabulary</u>** • Control group • Line of best fit • Observational study • Outliers • Random sample • Randomization • Regression • Sample size • Survey ASSESSMENT PROBLEMS S-IC.1 Basic • http://www.illustrativemathematics.org/illustrations/186 • http://www.illustrativemathematics.org/illustrations/122 • http://www.illustrativemathematics.org/illustrations/191 • http://www.illustrativemathematics.org/illustrations/123 • http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-1.html	*in five spins. If a truly fair coin produces five tails in five tosses 15 times out of 100 trials, then there is no reason to doubt the fairness of the coin. If, however, getting five tails in five spins occurred only once in 100 trials, then one could conclude that the coin is biased toward tails (if the coin in question actually landed five tails in five spins).* • *A powerful tool for developing statistical models is the use of simulations. This allows the students to visualize the model and apply their understanding of the statistical process.* • *Provide opportunities for students to clearly distinguish between a population parameter which is a constant, and a sample statistic which is a variable.* (ODE)		

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		S-IC.2 Basic • http://www.illustrativemathematics.org/illustrations/125 • http://www.illustrativemathematics.org/illustrations/244 • http://www.illustrativemathematics.org/illustrations/1099 • http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-2.html			
STATISTICS AND PROBABILITY **Making Inferences and Justifying Conclusions (S-IC)** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	M	**Students** S-IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **Major content** **Essential questions** • *How does randomization relate to sample surveys, experiments, and observational studies?* • *What is the difference between a control group and a treated group?* **Essential knowledge and skills** • Sample surveys, experiments and observational studies are three ways to collect data. • In an observational study, assignment of subjects into a treated group versus a control group is outside the control of the investigator. • In an observational study, the randomization is inherent in the population. • In controlled experiments, each subject is randomly assigned to a treated group or a control group before the start of the treatment. **Teaching Examples** In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. • Students should be able to explain techniques/applications for randomly selecting study subjects from a population and how those **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	TEACHER NOTES See instructional strategies in the introduction • *This cluster is designed to bring the four-step statistical process (GAISE model) to life and help students understand how statistical decisions are made. The mastery of this cluster is fundamental to the goal of creating a statistically literate citizenry. Students will need to use all of the data analysis, statistics, and probability concepts covered to date to develop a deeper understanding of inferential reasoning.* • *Students learn to devise plans for collecting data through the three primary methods of data production: surveys, observational studies, and experiments. Randomization plays various key roles in these methods. Emphasize that randomization is not a haphazard procedure, and that it requires careful implementation to avoid biasing the analysis. In surveys, the sample selected from a population needs to be representative; taking a*	RESOURCE NOTES See resources in the introduction • TI-83/84 and TI emulator	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
		techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment. • In statistics, an observational study draws inferences about the possible effect of a treatment on subjects, where the assignment of subjects into a treated group versus a control group is outside the control of the investigator (for example, observing data on academic achievement and socio-economic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment. (TUSD)	*random sample is generally what is done to satisfy this requirement. In observational studies, the sample needs to be representative of the population as a whole to enable generalization from sample to population. The best way to satisfy this is to use random selection in choosing the sample. In comparative experiments between two groups, random assignment of the treatments to the subjects is essential to avoid damaging problems when separating the effects of the treatments from the effects of some other variable, called confounding. In many cases, it takes a lot of thought to be sure that the method of randomization correctly produces data that will reflect that which is being analyzed. For example, in a two-treatment randomized experiment in which it is desired to have the same number of subjects in each treatment group, having each subject toss a coin where Heads assigns the subject to treatment A and Tails assigned the subject to treatment B will not produce the desired random assignment of equal-size groups.* • *The advantage that experiments have over surveys and observational studies is that one can establish causality with experiments.*		
	M	S-IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **Major content** **Essential questions** • *How do you use data from sample surveys, observational studies, and experiments to draw inferences and conclusions?* **Essential knowledge and skills** • A sample survey allows you to collect data from a subset of the population, and draw inferences about the larger population. • In a sample survey it is important to collect data from a random sampling that mimics the larger population. • Data from a sample survey can be used to estimate a population mean or proportion and then develop a margin of error from a simulation model. **Teaching Examples** • For S-IC.4 and 5, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness. • Students may use computer-generated simulation models based upon the results of sample surveys to estimate population statistics and margins of error. (TUSD) **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure			

CATEGORIES, DOMAINS, CLUSTERS	UNIT	STANDARDS/BENCHMARKS North Smithfield School Department	INSTRUCTIONAL STRATEGIES	RESOURCES	ASSESSMENTS
	M	S-IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **Major content** **Essential questions** • *Why do we use simulations to support inferences about data?* **Essential knowledge and skills** • Simulations of random samplings and experiments can be used to support inferences from the data. • Data from a randomized experiment can be used to compare two treatments. **Teaching Examples** • Treatment is a term used in the context of an experimental design to refer to any prescribed combination of values of explanatory variables. For example, one wants to determine the effectiveness of weed killer. Two equal parcels of land in a neighborhood are treated, one with a placebo and one with weed killer, to determine whether there is a significant difference in effectiveness in eliminating weeds. (TUSD) **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	• *Standard 4 addresses estimation of the population proportion parameter and the population mean parameter. Data need not come from just a survey to cover this topic. A margin-of-error formula cannot be developed through simulation, but students can discover that as the sample size is increased, the empirical distribution of the sample proportion and the sample mean tend toward a certain shape (the Normal distribution), and the standard error of the statistics decreases (i.e. the variation) in the models becomes smaller. The actual formulas will need to be stated.* • *Standard 5 addresses testing whether some characteristic of two paired or independent groups is the same or different by the use of resampling techniques. Conclusions are based on the concept of p-value. Resampling procedures can begin by hand but typically will require technology to gather enough observations for which a p-value calculation will be meaningful.* (ODE)		
	M	S-IC.6 Evaluate reports based on data. **Major content** a. Fit a function to the data; use functions fitted to data to solve problems **Essential questions** • *How can you determine if a report is showing you misleading data or conclusions?* • *When is it appropriate to use a randomized experiment as opposed to a sample survey?* **Essential knowledge and skills** • Reported data may be misleading due to, for example, sample size, biased survey sample, choice of interval scale, unlabeled scale, uneven scale, and outliers. **Teaching Examples** • Explanations can include but are not limited to sample size, biased survey sample, interval scale, unlabeled scale, uneven scale, and outliers that distort the line-of-best-fit. In a pictogram the symbol scale used can also be a source of distortion. • As a strategy, collect reports published in the media and ask students to consider the source of the data, **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure			

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		the design of the study, and the way the data are analyzed and displayed. Example: • A reporter used the two data sets below to calculate the mean housing price in Arizona as \$629,000. Why is this calculation not representative of the typical housing price in Arizona? ○ King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000} ○ Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000} (TUSD) **Academic vocabulary** • Control group • Line of best fit • Observational study • Outliers • Random sample • Randomization • Regression • Sample size • Survey **Common Student Misconceptions** • Students often confuse control group and test group. From the term "control" they tend to think of a control group as the one the test manipulates. • There are difficulties with observational and experimental probability with lines and curves of fit. Students often want everyone's result to be identical. • Students often have difficulty in choosing or identifying a random sample. For example, students might survey only their friends, which is a biased sample of the school population. ASSESSMENT PROBLEMS S-IC.3 Basic • http://www.illustrativemathematics.org/illustrations/1029 • http://www.illustrativemathematics.org/illustrations/1100 • http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-3.html S-IC.4Basic • http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-4.html S-IC.5Basic • http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-5.html S-IC.6Basic • http://www.shmoop.com/common-core-standards/ccss-hs-s-ic-6.html			

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STATISTICS AND PROBABILITY **Using Probability to Make Decisions (S-MD)** Use probability to evaluate outcomes of decisions. Use **Mathematical Practices** to 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics ★ 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning		**Students** S-MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). . **Essential questions** • *Why is drawing by lot or using a random number generator a fair way to make decisions?* • *How can you determine if a game is fair?* **Essential knowledge and skills** • Probabilities can be used to make fair decisions. **Teaching Examples** Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results. A game is fair if all players have an equal chance of winning. For more complicated games, it is often useful to calculate the expected value of the game (i.e., average winnings) for each player. Students begin to work with expected values in middle school. Examples: • John has designed a game using 2 dice. The rules state that Player A will get ten points if after rolling the dice the product is prime. Player B will get one point if the product is not prime. John feels this scoring system is reasonable because there are many more ways to get a non-prime product. Is John's game fair? Explain why or why not. • Suppose that a blood test indicates the presence of a particular disease 97% of the time when the disease is actually present. The same test gives false positive results 0.25% of the time. Suppose that one percent of the population actually has the disease. Suppose your blood test is positive. How likely is it that you actually have the disease? (TUSD) **Mathematical Practices** • Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure S-MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). **Essential questions** • *How can you evaluate the results of a diagnostic test?* **Mathematical Practices** • Make sense of problems and	TEACHER NOTES See instructional strategies in the introduction *Include more complex situations* • *This domain and cluster belong to STEM, and hence need not be for all students.* • *A game of chance is said to be fair if the expected net winnings are 0. If the expected net winnings is negative, then the player needs to decide if the game is worth playing. For example, a spinner has 18 red, 18 black and 2 green sections. Suppose, players gain a one score point if the spinner lands on red, otherwise the players loose a one score point. The probability the spinner lands on red is* $\frac{18}{38}$ • *The probability it lands elsewhere is* $\frac{20}{38}$ *So, the expected probability is* $1\times\frac{18}{38}+(-1)\times\frac{20}{38}= -.053$ *score points. This means that players should expect to lose a little over .05 of a score point every time they play the game. Calculating an expected value enables players to decide whether or not the game is worth playing* • *Expected values may be used to decide between two strategies. For example, suppose shop owner needs to decide whether to stock product A or product B and*	RESOURCE NOTES See resources in the introduction	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

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		• *Give an example of an event in which probabilities are used to make a decision, and explain how the probability is used.* **Essential knowledge and skills** • Probabilities can be used to analyze and evaluate decisions and strategies **Teaching Examples** Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results. A game is fair if all players have an equal chance of winning. For more complicated games, it is often useful to calculate the expected value of the game (i.e., average winnings) for each player. Students begin to work with expected values in middle school. Examples: • (The Monty Hall problem) Suppose you're on Let's Make a Deal, and you're playing the big deal of the day: you are given the choice of three curtains. Behind one curtain is a new car; behind the other two are zonks. You pick curtain number 1. The host, who knows where the car is, opens curtain number 3, which has a zonk. The host then says, "Do you want to switch curtains?" Is it better to switch or to keep your first choice, and why? • Wanda, the Channel 1 weather person, said there was a 30% chance of rain on Saturday and a 30% chance of rain on Sunday. It rained both days, and Wanda's station manager is wondering if she should fire Wanda. ○ Suppose Wanda's calculations were correct and there was a 30% chance of rain each day. What was the probability that there would be rain on both days? ○ Do you think Wanda should be fired? Why or why not? ○ Wanda is working on her predictions for the next few days. She calculates that there is a 20% chance of rain on Monday and a 20% chance of rain on Tuesday. If she is correct, what is the probability that it will rain on at least one of these days?	persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics ★ • Use appropriate tools strategically • Attend to precision • Look for and make use of structure	*can only stock one of them. Profit margins for A follow the distribution (in thousands of dollars): 5,4,3,2,1 with probabilities .1,.45,.3,.1,.05, respectfully. Those for B follow: 8,7,6,5,4,3,2,1,0 with probabilities: .1,.15,.15,.1,.1,0,0,0,.4. The expected profit by stocking A is 5(.1)+4(.45)+3(.3)+2(.1)+1(.05) = 3.45 thousands of dollars. The expected profit by stocking B is 8(.1)+7(.15)+6(.15)+5(.1)+4(.1)+0(.4) = 3.65 thousands of dollars. So, based on expected values of profit margins, the better choice would be to stock product B.* • *Conditional probabilities are situations where the interpretation of an observation is dependent upon or "conditioned on" some other factor. For example, a blood test has been shown to indicate the presence of a particular disease 95% of the time when the disease is actually present. The same blood test gives a false positive result 0.5% of the time. A false positive result suggests that even though the blood test indicates that the person has the disease (the positive part) but subsequent, additional testing indicates the person does not have that disease (hence positive but false or a false positive).* • *Suppose that one percent of the population actually has*		

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		• From: Connected Mathematics, "What Do You Expect?" (TUSD) **Academic vocabulary** • Expected value • Fair games • • False negative • False positive • Least-squares regression • Random number generator **Common Student Misconceptions** • Some students question whether the numbers produced by computer random number generators are truly random. The idea of a complicated but ordered algorithm making a random number generator is paradoxical. In fact, this is a valid concern, but a good random number generator will generate a sequence of numbers that passes statistical tests for randomness and has a very long cycle before it repeats. • Students are confused by the concepts of false positives and false negatives. For example, students will believe that a positive test result must always indicate that the person tested is likely to have the disease. However, if the actual incidence of the disease is low and the test can produce false positives, then it can happen that most of those who test positive actually don't have the disease. (TUSD) ASSESSMENT PROBLEMS S-MD.6 Basic • http://www.shmoop.com/common-core-standards/ccss-hs-s-md-6.html S-MD.7 Basic • http://www.illustrativemathematics.org/illustrations/1197 • http://www.shmoop.com/common-core-standards/ccss-hs-s-md-6.html	*the disease. If a person's blood test is positive, how likely is it that the person has the disease?* • *This scenario can be restated as the following conditional probability problem:* • *"What is the probability that a person actually has the disease given that (or conditioned on) the blood test indicates the person has the disease?"* • *There are two possibilities for a person to produce a positive blood test result: the person has the disease or the person does not have the disease.* (ODE)		
6. MODELING ★ 6.1 Choosing and using appropriate mathematics and statistics to analyze empirical situations		**Students** 6.1.1 Understand and use **descriptive modeling** which simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model - for example, graphs of global temperature and atmospheric CO_2 over time. 6.1.2 Understand that **analytical modeling** seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanics such as pollution or starvation intervene) follows a constant reproduction rate. Functions are an important tool for analyzing such problems. 6.1.3 Use graphing utilities, spreadsheets, computer algebra systems, and dynamic	TEACHER NOTES See instructional strategies in the introduction	RESOURCE NOTES See resources in the introduction	ASSESSMENT NOTES REQUIRED COMMON ASSESSMENTS • MID-TERM EXAM • FINAL EXAM • COMMON PROBLEMS/UNITS SUGGESTED FORMATIVE/ SUMMATIVE ASSESSMENTS See assessments in the introduction

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		geometry software as powerful tools that can be used to model purely mathematical phenomena (e.g. the behavior of polynomials) as well as physical phenomena. 6.1.4 Understands and use the basic **modeling cycle** ★: • **Problem:** Identifying variables in the situation and selecting those that represent essential features • **Formulate**: formulating a model by creating and selecting geometric, graphical, tabular, algebraic or statistical representations that describe relationships between the variables • **Compute:** analyzing and performing operations on these relationships to draw conclusions • **Interpret**: interpreting the results of the mathematics in terms of the original situation • **Validate:** validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable • **Report**: reporting on the conclusions and the reasoning behind them.